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ZIWET (A.) and FIELD (P.) *Introduction to Analytic Mechanics.* Macmillan, 1912. 12mo. 9 + 378 pp. $1.60.
ZOBEL (O. J.) See INGERSOLL (L. R.).

N. B.—Reprints of the Report given above may be procured, by members of the Association, on application to Professor W. D. CAIRNS, 27 King Street, Oberlin, Ohio.

REPORT OF THE NATIONAL COMMITTEE ON MATHEMATICAL REQUIREMENTS.

This Committee was appointed in 1916, by the Mathematical Association of America, with Professor J. W. YOUNG of Dartmouth College as chairman. Progress reports were published in this MONTHLY for October, 1916 and December, 1917. In the latter it was indicated that the following topics had been chosen for study: "The valid aims and purposes of mathematical study"; "Recent criticisms of mathematics"; "Formal discipline and the transfer of training"; "Desirable topics of algebra and their treatment"; "Scientific investigations"; "Questionnaires"; "World experience as to mathematical curricula and the training of teachers." Other topics for investigation have since been decided upon. The extraordinary importance of an exhaustive national inquiry in connection with such questions can hardly be overestimated. It is therefore with great satisfaction that the Committee announces that funds, sufficient for carrying on its work in a very thorough manner, have been secured.

At a meeting held in New York on May 22, 1919, the General Education Board appropriated the sum of $16,000 for the use of the National Committee. The plan involves the appointment of one college man and one high school man

[1] Taken over and issued by the Yale University Press in 1913.

to devote their full time to the work of the Committee, their salaries for one year to be paid out of the fund mentioned. There is also ample provision for stenographic and other clerical help, and for printing, stationery, and postage, as well as for travelling expenses. It is hoped that a more extended account of the proposed work of the Committee published in the next issue of the MONTHLY. By that time, also, the organization of the Committee under the new plan should have been effected.

FOURTH ANNUAL MEETING OF THE OHIO SECTION.

The fourth annual meeting of the Ohio Section of the Mathematical Association of America was held at the Ohio State University, Columbus, on Friday, April 18, 1919, in connection with the meetings of the Ohio College Association. Professor C. N. Moore occupied the chair, being relieved by Professor K. D. Swartzel for an interval.

The following thirty-eight persons were registered, all but the last ten being members of the Association:

R. B. Allen, Kenyon College; W. E. Anderson, Miami University; G. N. Armstrong, Ohio Wesleyan University; C. L. Arnold, Ohio State University; C. B. Austin, Ohio Wesleyan University; Grace M. Bareis, Ohio State University; Mrs. W. E. Beckwith, Western Reserve University; R. D. Bohannan, Ohio State University; R. L. Borger, Ohio University; J. B. Brandeberry, Toledo University; A. G. Caris, Defiance College; T. M. Focke, Case School of Applied Science; M. E. Graber, Heidelberg University; Harris Hancock, University of Cincinnati; William Hoover, Columbus; Emma L. Konantz, Ohio Wesleyan University; H. W. Kuhn, Ohio State University; C. N. Moore, University of Cincinnati; C. C. Morris, Ohio State University; Anna H. Palmié, Western Reserve College for Women; S. E. Rasor, Ohio State University; Hortense Rickard, Ohio State University; W. G. Simon, Adelbert College; S. A. Singer, Capital University; K. D. Swartzel, Ohio State University; R. B. Wildermuth, Capital University; F. B. Wiley, Denison University; D. T. Wilson, Case School of Applied Science. Non-members: H. M. Beatty, Ohio State University; W. S. Beckwith, Ohio Northern University; Clara F. Brumbach, Denison University; C. A. Hahn, Otterbein College; G. W. McCoard, Ohio State University; F. W. Parsons, Ohio Northern University; Anna B. Peckham, Denison University; F. J. Shollenberger, Mt. Union College; C. H. Skinner, Ohio Wesleyan University; S. E. Slocum, University of Cincinnati.

The following program, slightly changed in order, was carried out as arranged by the executive committee:

General theme: Mathematics and Warfare.

1. Chairman's Address: The rôle of mathematics in world progress by Professor C. N. MOORE, University of Cincinnati.

2. The mathematical features of navigation by Professor D. T. WILSON, Case School of Applied Science.
3. Discussion of the preceding by Professor W. E. ANDERSON, Miami University.

Business and Intermission.

4. Ballistics as applied mathematics by Professor M. E. GRABER, Heidelberg University.
5. The mathematics of aviation by Professor S. E. SLOCUM, University of Cincinnati. (Introduced by Professor C. N. Moore.)

About twenty of those present dined together at the Ohio Union in the evening. A recess was taken during the evening program to attend the lecture given before the coöperating societies by the representative of the Carnegie Foundation, on "The Pension Problem for College Teachers." Many remained over Saturday to attend meetings of the Ohio College Association.

ABSTRACT OF PAPERS.

1. In speaking of the rôle of mathematics in world progress, Professor Moore said that the great importance of mathematics in various war activities was due to its extreme usefulness in many technical and scientific labors. Hence, the war record of mathematics shows in general that knowledge of this subject is of equal value in connection with the tasks of peace. In addition to this, a number of uses of mathematics during the war, such as the application of mathematical analysis to the study of wounds, may be regarded as positive contributions to world progress. This application of mathematics, together with other recent applications in such fields as biology, sociology and economics, illustrates the fact that the domain of usefulness of mathematics is as wide as the domain of science. It is the duty of mathematicians to make some effort to bring this fact home to the general public and to strive to make wider use of the applications of mathematics in connection with their courses. In this way many more college students will be induced to continue their mathematical studies through a first course in calculus, and the better trained workers in other fields will thus be able to discover many new applications of mathematics.

2. Mathematics is essential to the navigator in determining the angle of departure, in getting the details of his course, and in the maintenance of his course. The track is usually either a loxodrome or a great circle. For the loxodrome the angle of departure and distance between ports are computed from the principles of the Mercator projections. For a great circle these are computed from spherical triangles of which the vertices are the two ports, the pole and the vertex of the arc. In order to maintain his course, the navigator must be able to determine his position at sea by the principles of practical astronomy.

3. Professor Anderson noted that the principles enunciated in the theory of navigation may be utilized in creating an interest in the applications of mathematics, and thus in mathematics itself. The astronomical principles should be used as a means of creating an interest in astronomy, which should be made

available to the great majority of our students. The apparently complex formulæ of spherical trigonometry can be shown to be reducible to three simple formulæ, thus illustrating the fact that the fabric of mathematical structure is exceedingly simple, failure to grasp which fact is the source of much of the student's difficulty in his mathematical work.

4. In this paper the mathematical foundations of interior and exterior ballistics were outlined by Professor Graber. The applications of mathematics to the theory of explosives, principal stresses and strains and the design of artillery were mentioned and a typical case of design considered. Sarau's modified formulae of design were given and their application indicated. Then followed a comparison of the Mayevski velocity-resistance graph and the corresponding graph embodying the latest French results. An analytic expression was exhibited for the velocity-resistance relation between the velocities 1700 ft./sec. and 700 ft./sec. In the field of rational exterior ballistics, Siacci's method was developed and a graphic solution of problems in direct fire explained from constructed graphs. Graphs of different trajectories were also exhibited with their corresponding calculated elements.

5. The purpose of the paper by Professor Slocum was to call attention to the possibilities of aëronautics as a field of mathematical research. Thirty years ago there were authoritative proofs that mechanical flight was impossible, and it was only fifteen years ago that this was disproved by actual demonstration. In this brief period, aviation has attained a remarkable practical development and its theoretical development must follow in the near future. The present content of aëronautics was outlined under three heads: experimental, theoretical, and structural. In the second of these, the extent of mathematical analysis was indicated, and the need for an extension of theory to supplement practice pointed out. Attention was called to the future possibilities of aviation, and the duty and privilege of mathematicians to aid in the conquest of the air.

At the Friday evening round table, held in the Ohio Union and attended by twenty-three persons, the discussion, widely participated in, centered upon experience and lessons from S. A. T. C. work, supervised study, and war activities. Professor Swartzel spoke of his experiences as an inspector of S. A. T. C. work. Professor Hancock gave results of extensive mathematics tests given S. A. T. C. applicants at the University of Cincinnati, which indicated very inadequate preparation on the part of the men.

The following were elected as officers for 1919–20:

Chairman: R. L. Borger, Ohio University.
Secretary-Treasurer: G. N. Armstrong, Ohio Wesleyan University.
Third member of the Executive Committee: S. E. Rasor, Ohio State University.

G. N. Armstrong, *Secretary-Treasurer.*

THE APRIL MEETING OF THE ROCKY MOUNTAIN SECTION.

The Rocky Mountain Section of the Mathematical Association of America held its annual meeting at Ft. Collins, Colorado, April 12, 1919; the members were royally entertained by President Lory and the professors of Colorado Agricultural College. The papers were extremely interesting and were discussed very thoroughly. The twenty-four persons in attendance included the following thirteen members of the Association: I. M. DeLong, University of Colorado, J. C. Fitterer, University of Wyoming, W. H. Hill, High School, Greeley, Col., Claribel Kendall, University of Colorado, O. C. Lester, University of Colorado, G. H. Light, University of Colorado, S. L. Macdonald, Colorado Agricultural College, O. A. Randolph, University of Colorado, C. B. Ridgaway, University of Wyoming, C. H. Sisam, Colorado College, C. S. Sperry, University of Colorado, T. O. Walton, Colorado School of Mines, and J. W. Woodrow, University of Colorado.

The following papers were read at morning and afternoon sessions:

Generalization of the addition formulæ in trigonometry by Professor I. M. DeLong, University of Colorado; New experiments for the laboratory course in general physics, and Positions and properties of the cardinal points of a lens system from the standpoint of the wave theory, by Professor J. W. Woodrow, University of Colorado; Probable errors of Mendelian class frequencies by Mr. Breeze Boyack, Colorado A. and M. College; Calculation of high frequency resistance of wires by Professor O. C. Lester, University of Colorado; Development of empirical formulas for the solution of problems in hydraulics by Mr. L. R. Parshall, Colorado A. and M. College; Electromagnetic waves on wires by Professor O. A. Randolph, University of Colorado; On non-ruled optic surfaces whose plane sections are elliptical by Professor C. H. Sisam, Colorado College; Some characteristics of the mercury arc by Professor J. W. Woodrow, University of Colorado; An application of hyperbolic functions of a complex quantity to the determination of the performance of long distance alternating current transmission lines by Professor L. S. Foetz, Colorado A. and M. College.

An invitation from Colorado College to hold the next meeting there was received and accepted. The following were elected officers for the ensuing year:

Chairman: C. H. Sisam, Colorado College.

Vice-chairman: T. O. Walton, Colorado School of Mines.

Secretary-Treasurer: G. H. Light, University of Colorado.

G. H. Light, *Secretary-Treasurer.*

ELECTION OF MEMBERS.

The Council of the Association has elected to membership the following persons and institutions:

To individual membership.

W. S. BECKWITH, A.M. (Harvard). Prof., Ohio Northern Univ., Ada, Ohio.
B. H. BROWN, A.M. (Brown). Instr., Harvard Univ., 1919–20.
MRS. THEODOSIA T. CALLAWAY, B.S. (Columbia). Prof., Stephens Coll., Columbia, Mo.
JESSIE R. CAMPBELL, A.B. (Syracuse). Instr., Hollywood Junior Coll., Los Angeles, Calif.
W. P. DOBSON, M.A.Sc. (Toronto). Lab. Engr., Hydro Elec. Power Commission of Ontario, Toronto, Can.
PHILIP FRANKLIN, B.S. (Coll. of the City of New York). Ballistic Computer, Ord. Dept., Aberdeen Proving Ground, Md.
ELIZABETH FREAS, A.B. (Lake Erie). Computer, Coast and Geod. Survey, Washington, D. C.
J. S. GEORGES. Student, Maryville Coll., Maryville, Tenn.
ADELE C. HOLTWICK, A.M. (Washburn). Instr., Henry Kendall Coll., Tulsa, Okla.
ROBERT MASSEY, B.A. (Queen's Univ., Belfast). Prin., Prevocational Sch., Calgary, Alb., Canada.
REV. L. F. OTT, S.J. Prof. Canisius Coll., Buffalo, N. Y.
F. W. PARSONS, B.S. in E. E. (Ohio Northern). Asst., Ohio Northern Univ., Ada, Ohio.
ANNA B. PECKHAM, A.M. (Denison). Asso. Prof., Denison Univ., Granville, Ohio.
R. V. PRITCHARD, B.S. (Centr. Normal Coll.) Instr., Case Sch. of Applied Sc., Cleveland, Ohio.
REV. PATRICK RAFFERTY, S.J. Prof., Coll. of the Holy Cross, Worcester, Mass.
J. C. ROGERS, B.S. (Earlham). Dean, Piedmont Coll., Demorest, Ga.
MAX SHAMOS. Engr., Cleveland, Ohio.
MARGARETHE M. SMITH, A.M. (Radcliffe). Instr., Wilson Coll., Chambersburg, Pa.
EUGENE STEPHENS, St. Louis, Mo.
J. H. TANNER, Ph.D. (New Hampshire Coll.). Prof., Cornell Univ., Ithaca, N. Y.
A. P. WHISENHUNT, A. M. (Lenoir; Catawba). Head of Dept. of Math., Catawba Coll., Newton, N. C.

To institutional membership (with the official representative).

UNIVERSITY OF ARIZONA, Tucson, Ariz. Pres. R. B. von Kleinsmid.
ST. OLAF COLLEGE, Northfield, Minn. Prof. P. G. Schmidt.
ADELPHI COLLEGE, Brooklyn, N. Y. Dr. F. D. Blodgett.
CANISIUS COLLEGE, Buffalo, N. Y. Pres. M. J. Ahern.
BETHANY COLLEGE, Bethany, W. Va. Prof. E. H. Vance.

W. D. CAIRNS, *Secretary-Treasurer.*

QUESTIONS AND DISCUSSIONS.

EDITED BY W. A. HURWITZ, Cornell University, Ithaca, N. Y.

REPLIES.

21. For the diophantine equation

$$x^3 - y^3 = 17$$

there are known the following solutions:

$$\begin{array}{lrrrrrrrr} x = & 3, & 4, & 5, & 9, & 23, & 282, & 375, & 378661, \\ y = & -2, & -1, & 2, & 4, & 8, & 43, & 52, & 5234. \end{array}$$

One of our readers, who supplied the foregoing facts, desires to know the answers to the following questions: Are there other solutions of the given diophantine equation?[1] How may all the solutions of this equation be found by a systematic procedure?

REPLY BY E. B. ESCOTT, Chicago, Ill.

A complete solution of questions of this kind does not seem to be possible. In default of a complete solution, I give four methods by which all of the given solutions may be found.

I. Let $x = a$, $y = b$ be a solution found by trial. From

$$x^3 - y^3 = 17, \tag{1}$$

and

$$a^3 - b^3 = 17, \tag{2}$$

we get

$$(x + a)(x - a) = (y - b)(y^2 + by + b^2). \tag{3}$$

Let $\dfrac{x - a}{y - b} = m$; or $x = m(y - b) + a$.

Substituting in (3), removing factor $y - b$ from both members and arranging,

$$y^2 - (m^2 - b)y + bm^2 - 2am + b^2 = 0. \tag{4}$$

The roots of (4) will be rational if its discriminant is a perfect square, i.e., if

$$m^4 - 6bm^2 + 8am - 3b^2 = r^2. \tag{5}$$

Solutions of this equation may be found by the following method, given by Euler in his *Elements of Algebra*, part II, Chapters 8 and 9.

For example, let $a = 3$, $b = -2$; then (5) becomes

$$m^4 + 12m^2 + 24m - 12 = r^2. \tag{6}$$

By trial, we find that $m = 1$ is a solution.

Let $m = p + 1$; then

$$p^4 + 4p^3 + 18p^2 + 52p + 25 = r^2. \tag{7}$$

Let

$$r = ep^2 + fp + g.$$

Substituting in (7) and combining terms

$$(e^2 - 1)p^4 + (2ef - 4)p^3 + (2eg + f^2 - 18)p^2 + (2fg - 52)p + (g^2 - 25) = 0. \tag{8}$$

In (8) we may choose e, f, and g so that the equation reduces to one of the first degree in p.

(*a*) If $e^2 - 1 = 0$, $2ef - 4 = 0$, $2eg + f^2 - 18 = 0$, we have $e = 1, f = 2, g = 7$.

Then

$$p = 1, \text{ and } m = 2.$$

(*b*) If $e^2 - 1 = 0$, $2ef - 4 = 0$, $g^2 - 25 = 0$, we have $e = 1, f = 2, g = \pm 5$.

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[1] In *Mathematical Questions and Solutions from the "Educational Times,"* new series, Vol. 8, 1905, p. 53, R. F. Davis gives all of the solutions indicated above and purports to show that "any single solution leads to an interminable chain of other solutions." The question is also discussed by A. Cunningham, *Mathematical Questions*, etc., *l.c.*, p. 54; and vol. 14, 1908, pp. 107–108. See also *L'Interméd. des Math.*, 1905–13, Qu. 2512, 4057.—Editor-in-Chief.

Then

$$p = -8, \text{ and } -3; \text{ and } m = -7, \text{ and } -2.$$

(c) If $e^2 - 1 = 0$, $2fg - 52 = 0$, $g^2 - 25 = 0$, we have $e = 1$, $f = \pm 26/5$, $g = \pm 5$.
Then

$$p = -119/40, \text{ and } -1/15; \text{ and } m = -79/40, \text{ and } 14/15.$$

(d) If $2eg + f^2 - 18 = 0$, $2fg - 52 = 0$, $g^2 - 25 = 0$, we have $e = -113/125$, $f = 26/5$, $g = 5$.

Then $p = -8725/119$, and $m = -8606/119$.

Substituting these values of m in (4), we get the following solutions:

m	y		x	
2	8	−2	23	3
−7	52	−1	−375	−4
−2	4	2	−9	−5
$-\frac{79}{40}$	$\frac{137}{64}$	$\frac{94}{25}$	$-\frac{2651}{512}$	$-\frac{1047}{125}$
$\frac{14}{15}$	$-\frac{8}{9}$	$\frac{94}{25}$	$\frac{109}{27}$	$\frac{1047}{125}$

Since one solution just found is $x = 5$, $y = 2$, we can take $a = 5$, $b = 2$.

Substitute these values in (5).

$$m^4 - 12m^2 + 40m - 12 = r^2. \tag{9}$$

Since $m = 2$ is a solution, let $m = p + 2$ and proceed as before. We find the following values of m, and the corresponding solutions:

m	y		x	
2	4	−2	9	−3
3	8	−1	23	−4
−7	4	43	−9	−282

Similarly, putting $a = 375$, $b = 52$, we find $m = 73$; whence $y = 43$, and 5234;
$x = -282$, and 378661.

The above method will give an indefinite number of rational solutions but apparently only a limited humber of integral solutions.

II. Consider the curve of the third degree,

$$x^3 - y^3 = 17. \tag{1}$$

The tangent at (a, b) is

$$2ax - 3b^2y + a^3 - 51 = 0. \tag{2}$$

This cuts the curve again in the point

$$x = -\frac{(a^3 + 153)^2 - 31212}{8a^3}, \qquad y = \frac{b}{4a^3}(a^3 - 153). \tag{3}$$

Taking any rational solution (a, b) of (1), we get a second rational solution from (3). From this solution we get another, etc.

Example. From $a = 3$, $b = -2$, we get from (3)

$$x = 23, \quad y = 8.$$

III. Under the first method, equation (4) is a quadratic equation in y. Calling the roots y_1 and y_2 we have

$$y_1 + y_2 = m^2 - b.$$

If we have for a given value of m, one value of y, the other value may be found from this relation.

Considering the same equation as a quadratic in m, and calling the roots m_1 and m_2 we have

$$m_1 + m_2 = -\frac{2a}{y - b}.$$

In the same way given a value of y and one value of m, we can find another value of m.

In this way we can find very rapidly a chain of values of m and y.

Example: Let $a = 3$, $b = -2$.

$$y_1 + y_2 = m^2 + 2, \tag{1}$$

$$m_1 + m_2 = -\frac{6}{y+2}. \tag{2}$$

Starting with $y = -2$, $m = 2$, (1) gives $y = 8$,
(2) gives $m = \infty$;
$y = 8$, $m = 2$, (2) gives $m = -\frac{1}{3}^3$;
$y = 8$, $m = -\frac{1}{3}^3$, (1) gives $y = \frac{1}{2}^3$;
$y = \frac{1}{2}^3$, $m = -\frac{1}{5}^3$, (2) gives $m = -\frac{4}{1}^9$;
. . . . . .

IV. Solutions of $x^2 - y^3 = 17$ in integers may be found by Gauss's "Method of Exclusion" by which *all* the solutions under any desired limit may be found.

Since $$y^3 + 17 = x^2,$$

the residues of $y^3 + 17$ to any modulus must be quadratic residues. Therefore, we can exclude all values of y for which $y^3 + 17 \equiv$ a non-quadratic residue. This gives us the following:

$y \not\equiv 0 \pmod 3$;
$y \not\equiv 1, 5 \pmod 8$, $y \not\equiv 7 \pmod 8$, excepting $y \equiv 15 \pmod{16}$;
$y \not\equiv 0, 1 \pmod 5$, $y \not\equiv 2 \pmod 5$, excepting $y \equiv 2 \pmod{25}$;
$y \not\equiv 0 \pmod 7$;
$y \not\equiv 0, 1, 5, 6, 7 \pmod{11}$, $y \not\equiv 3 \pmod{11}$, excepting $y \equiv -30 \pmod{121}$;
$y \not\equiv 1, 3, 9 \pmod{13}$;
$y \not\equiv 0, 3, 5, 6, 7, 10, 11, 12, 14 \pmod{17}$;
$y \not\equiv 1, 7, 10, 11, 13, 15 \pmod{19}$;
. . . . . .

Rejecting all the cases above and others (mod 23 and 29) we find that the only possible solutions less than 1,000 are

$$y = 2, 4, 8, 43, 52, 934, 988.$$

Trying these values, we find the first five give solutions. The last two do not.

Remark. It is worth while noting the essential difference between the equations

$$x^2 - a = y^3,$$

and

$$x^2 + a = y^3.$$

The latter equation has only a finite number of solutions, which can be found by a perfectly general method,[1] but it is quite different with the first equation. The reasen is that the properties of quadratic forms with positive determinant are quite different from those with a negative determinant. (The word "determinant" is used in its original sense of discriminant.)

NOTE. In the last section of an article[2] entitled "The Diophantine Equation $y^2 - k = x^3$," L. J. Mordell indicates the results obtained as to the existence of an infinite number of solutions, a finite number, or none, for all values of k between ± 100. In accordance with his list, the case $k = 17$ should yield an infinite set of integral solutions. The question under discussion thus still presents as a challenge to our readers the alternative of either disproving Mr. Mordell's result for the case $k = 17$, or else verifying it and producing one or more new solutions.—EDITOR.

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[1] Gerono showed (*Nouvelles Annales de Mathématiques*, 1877, pp. 325–326) that there are *no* integral solutions of the equation $x^2 + 17 = y^3$.—Editor-in-Chief.

[2] *Proceedings of the London Mathematical Society*, series 2, vol. 13 (1914), pp. 60–80.

32. In a discussion of the Peaucellier[1] Cell by analytic methods the following equations are obtained:

$$(1)\ (x_2 - x_1)^2 + (y_2 - y_1)^2 - b^2 = 0; \qquad (2)\ (x_3 - x_1)^2 + (y_3 - y_1)^2 - b^2 = 0;$$

$$(3)\ (x_2 - X)^2 + (y_2 - Y)^2 - b^2 = 0; \qquad (4)\ (x_3 - X)^2 + (y_3 - Y)^2 - b^2 = 0;$$

$$(5)\ x_2^2 + y_2^2 - K^2 = 0; \qquad (6)\ x_3^2 + y_3^2 - K^2 = 0;$$

$$(7)\ x_1^2 + y_1^2 - 2cx_1 = 0.$$

The result of eliminating $x_1, y_1, x_2, y_2, x_3, y_3$ gives an equation of the first degree, which establishes that the linkage will trace a straight line. There are various ways of effecting this elimination.

1. What element of the situation is left unused by the following procedure in the elimination?

(*a*) From equations (1), (3), (5) eliminate x_2 and y_2 and obtain an equation

$$(8)\ f_1(x_1, y_1) = 0.$$

(*b*) From equations (2), (4), (6) eliminate x_3 and y_3 and obtain an equation

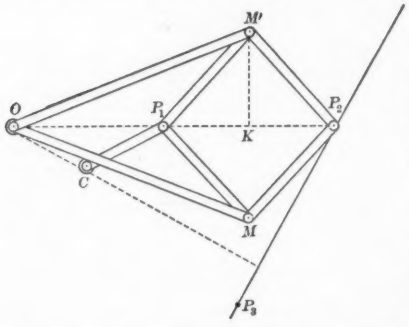
$$(9)\ f_2(x_1, y_1) = 0.$$

(*c*) From equations (7), (8), (9) eliminate x_1 and y_1 and obtain the desired equation.

2. How should this procedure be supplemented to secure the result?

Reply by G. H. Ling, University of Saskatchewan.

The configuration discussed has a line of symmetry; the equations developed show that the points (x_2, y_2) and (x_3, y_3) are *either coincident or symmetrical with respect to the line of symmetry*, but they fail to express that the essential feature of the configuration is that the two points mentioned are *not* coincident.



The suggested procedure for elimination cannot take account of this unexpressed fact. Equations (8) and (9) are identical and the elimination of x_1 and y_1 from (7), (8) and (9) halts because of the lack of three independent equations.

The procedure may be supplemented as follows:

If two of the three equations (1), (3), (5) be solved for x_2 and y_2 two distinct solutions are obtained and these must be the values for x_2 and y_2, and for x_3 and y_3. Both of these sets of values must satisfy that one of the equations (1), (3), (5) which is not employed in the solution just mentioned. The substitution of both of these sets of values in this third equation yields two equations (8) and (9) which associated with the equation (7) yield a single resultant equation free from x_1 and y_1. This last equation is the equation of the desired locus. (See May Monthly, p. 188.)

It is easy to see that the case here discussed can be generalized, though in the more general cases the treatment of the problem would not be nearly so simple.

DISCUSSIONS.

Every teacher of trigonometry, analytic geometry, and the calculus has at times experienced the difficulty of inducing students to regard the radian measure of an angle as a pure number. How is this difficulty to be met? Professor Car-

[1] If reference is made to the article on "Linkages" in the December, 1915, Monthly, by Mr. Leavens, the following coördinates may be applied to his figure (reproduced above): O (0, 0); C $(c, 0)$; P_1 (x_1, y_1); $M(x_2, y_2)$; M' (x_3, y_3); P_2 (X, Y).

ver's discussion below, covering substantially the ground of a paper presented by him at the summer meeting of the Mathematical Association of America in September, 1918, should be suggestive and stimulating, and will, it is hoped, give rise to further expressions of opinion.

TRIGONOMETRIC FUNCTIONS—OF WHAT?[1]

BY W. B. CARVER, Cornell University.

The first idea which our students get of a trigonometric function—say $\sin x$—is that the argument x is an *angle*, a geometric entity. According to this conception, the sine of a right angle is 1 whether one thinks of it as an angle of $90°$ or of $\pi/2$ radians; while $\sin 2$ has different values according as the 2 means 2 radians, 2 degrees, or 2 right angles.[2] Later the student needs the notion of the number $\sin x$ as a function of the *number* x, a functional relation which is not dependent upon any sort of geometric ideas or units of geometric measurement. This second point of view is needed not only by those who specialize in mathematics, but also by the large class of students who go no further than a first course in the calculus, and whose purpose may be entirely utilitarian.

It is the writer's conviction that, certainly in our text-books, and possibly in our teaching, we are not doing as much as we might to help the student across from the one point of view to the other.

In our courses in trigonometry the first point of view must prevail: but the way may be prepared for the second by insisting upon familiarity with the circular measurement of angles. The radian unit should be introduced early and *used frequently* throughout the course. The tables of functions should have a column giving the angle in radians adjacent to the column reading degrees and minutes.[3] In the problems for solution in both right and oblique triangles, the given angles should, in at least a few cases, be expressed in radians. The relation of the number π to this method of measuring angles should be made clear. That there is confusion on this point is indicated by such questions as "Why does π mean 3.1416 in one place, and in another place $180°$?"

Analytic geometry should bring us nearer to the idea of a trigonometric function of a number. Should a student be permitted to draw the curve represented by the equation $y = \sin x$ with the wave-length and amplitude in any ratio that pleases him? If so, may he plot $4x^2 + y^2 = 1$ as a circle, or $x^2 + y^2 = 1$ as an ellipse? In drawing such trigonometric curves, should we not insist that the units of length on the x and y axes shall be the same, and that $\sin x$ means the sine of an angle of x radians? Queerly enough, in polar coördinates the trouble arises when we do *not* have trigonometric functions of θ rather than

[1] Read before the Mathematical Association of America, September 6, 1918.

[2] Many of the text-books state explicitly the convention that the radian is to be assumed when no other unit is indicated.

[3] Such tables are surprisingly scarce; as are also protractors reading radians and decimal parts of radians.

when we do. Why does the curve $\rho = \theta^2$ raise the question of the unit of angle measurement while the curve $\rho = \sin\theta$ does not? Is the θ of our polar coördinates a number or an angle? The plotting of the curve $\sin\rho = \theta$ presents the difficulty in even more acute form. The best way out of all these difficulties would seem to be to emphasize the fact that ρ and θ are numbers; and that in polar coördinates ρ is represented graphically by ρ units (any convenient unit) of length, while θ is represented by an angle of θ *radians*.

It is in the course in calculus that the student must be brought to a full realization of the significance of $\sin x$ as a function of a number. In deducing the formula $d\sin x/dx = \cos x$, the idea of x as an angle is still in evidence; but it should be emphasized that the validity of the formula depends upon the fact that x means the number of *radians* in the angle. One good method of impressing this point is *first* to deduce a formula assuming that x means an angle of x degrees, so that the student may see clearly the advantage of the use of the radian unit. Being thus committed to this unit of angle measurement, it will follow later that in the formula

$$\int \frac{dx}{\sqrt{1-x^2}} = \sin^{-1} x + C,$$

$\sin^{-1} x$ means the number of radians in the angle[1] whose sine is x. How many of our students have any reason for a choice between $\pi/4$ and $45°$ as a value for the integral $\int_0^1 \sqrt{1-x^2}dx$, other than the fact that the latter value does not seem to give the "right answer" for the area of the quarter circle?

When we have expanded $\sin x$ in a power series, convergent (happily) for all values of x, we have finally the basis for a definition of the function which is independent of geometric notions. An analogy may be helpful to the student at this point. The geometric notions of the area and length of side of a square may be used to exhibit the relation between the numbers x and $\sqrt{x}$; but nevertheless this number relation is independent of such geometric considerations. And the student of fairly keen mathematical insight will be interested to see that he now has a relation between the numbers x and $\sin x$ which is similarly independent of geometric considerations.

RECENT PUBLICATIONS.

REVIEWS.

MATHEMATICS FOR FRESHMEN.

1. *Introduction to the Elementary Functions.* By R. B. McLENON with the editorial coöperation of W. J. Rusk. Boston, Ginn, 1918. 8vo. 9 + 244 pp. Price $1.80.

[1] It is also important, of course, that $\sin^{-1} x$ should have been so defined as to make it a single-valued and continuous function.

2. *Freshman Mathematics.* By W. R. RANSOM. New York, Longmans, 1918. 12mo. 12 + 285 pp. Price $1.50.

3. *Elementary Mathematical Analysis. A textbook for first-year college students.* By C. S. SLICHTER. Second edition revised and entirely reset. New York, McGraw-Hill, 1918. 12mo. 18 + 497 pp. Price $2.50.

4. *Unified Mathematics.* By L. C. KARPINSKI, H. Y. BENEDICT, and J. W. CALHOUN. Boston, Heath, 1918. 12mo. 8 + 522 pp. Price $2.80.

What shall be the content of the course in freshman mathematics? This is a question that has been occupying the minds of many of the teachers of mathematics in our colleges for the past five or six years. The traditional course in mathematics for the freshman year has held sway for a long period of time and the assaults that have been directed against it as a required subject have, in the main, left it secure.

Why should the content and arrangement of the course be changed? The fact that its security might be at stake is neither a sufficient nor an intelligent reason. So long as technique is regarded as the all-important aspect of the subject, there is no reason why there should be any change from the traditional course, except possibly toward unification and even this may be questioned. On the other hand, in view of the fact that freshman mathematics is a required course in the majority of our colleges, there is a feeling that all the emphasis should not be put on technique, but that insight and understanding should receive their due share of emphasis, thus affording a clearer perception or recognition of the worth of the subject in practical application. The four books which we shall now examine attempt to answer the question confronting us.

1. The subject matter which consists of trigonometry, elementary analytic geometry, and an introduction to the differential calculus, is presented in ten chapters (230 pages). This does not include the appendix, containing nine pages of review work in algebra and geometry. The authors take functionality as the unifying principle and endeavor to keep this idea foremost throughout the book.

Chapters I and II deal with graphical representation and functional relations. These two chapters are largely introductory. Much of the material contained in the 32 pages of Chapter III on graphic algebra seems somewhat disconnected and rather elementary. The quadratic equation with which the student should be familiar is taken up in too great detail. It also seems a waste of time and space to devote three pages to the factor theorem for the quadratic equation, and an even greater waste to discuss in this chapter the maximum and minimum values of the quadratic function, which comes up again in the differential calculus. Chapters IV and IX consisting of 52 pages are devoted almost entirely to trigonometry, the solution of triangles by logarithms being deferred until the exponential and logarithmic functions are taken up in Chapter X. This separation of trigonometry into three parts seems rather artificial and causes one to wonder if functionality, which is the unifying principle, does not necessitate an arrangement somewhat unnatural and not conducive to the best results. There seems

to be no good reason why Chapters IV and IX should not be combined. In the beginning of Chapter IV the trigonometric functions are defined at once for any angle. In this chapter the work is well arranged. Although velocities and forces are considered under the application of the trigonometric functions, radian measure, and angular velocity are nowhere mentioned.

Chapter V deals with simple irrational functions and the locus problem, and the three following chapters are devoted to the consideration of the straight line, the circle and the conics. In Chapters VI and VII the geometric aspect of the work receives emphasis, while graphic algebra predominates in Chapter VIII.

Some disappointment is felt over the fact that in Chapter X, the logarithmic and exponential functions seem to be introduced solely for the use of logarithms in the solution of triangles. In our opinion not enough consideration is given to the functional relation $y = \log x$. The important functions $y = \log_e x$, $y = e^x$ and $y = e^{-x^2}$ are not mentioned. The concluding chapter of the book, which is on the differential calculus, presents in 32 pages, some necessary theorems on limits, the definition of the derivative of a function, rules for finding the derivative of some of the simpler functions, rates, and maxima and minima. There is also a well-chosen set of problems. The material of this chapter is well selected and clearly presented. In our opinion the notation for the derivative, $D_x y$, is most confusing to the beginner and for this reason should not be employed.

2. If the object of the freshman course in mathematics is to give the student interesting material with which to work and from which may be derived useful information, and if mathematical coherence and completeness are of no great importance to the freshman, except as demanded for such information, this book can be highly praised. But the well-selected material is not the only merit the book has. The work is well presented and the explanations are clear. It is quite surprising too, to find that so much can be put into 285 pages without undue congestion.

There are twenty-six short chapters in the book. The more elementary part of the work is given in the first seven chapters. The leading topics of these chapters are arithmetical computation, the application of trigonometry to right triangles, and an introduction to coördinates. The trigonometric functions are defined for acute angles and the definitions for any angle are not given when coördinates are first introduced but are deferred until the consideration of vectors comes up (Chapter XII) toward the close of the work on trigonometry. This causes an awkward situation on page 96 where the relation, $\sin A = \sin (180° - A)$, must be given as a definition.

The next seven chapters of the book are devoted entirely to trigonometry and its applications. The use of the slide rule is explained and a chapter on spherical trigonometry and its applications is added.

Determinants, variation, and the solution of numerical equations are the topics of algebra which are taken up in Chapters XIV–XVII.

The last eight chapters of the book deal primarily with differential and integral calculus and their applications. The derivative is made use of in plotting

exponential and other special curves; in the study of rates and in finding maximum and minimum values. The fifteen pages of the integral calculus are devoted to its application in finding areas and volumes.

The book on the whole is interesting and stimulating. The exercises and problems have been selected and graded with care, and no demand is made on the students' ability to handle complicated algebraic expressions. On the other hand, the book lacks coherence. The analytic geometry that is given is scattered throughout the book, and one has a feeling that this very important subject has not received the attention it merits. The conic sections are inadequately treated and many of their important properties are not mentioned. Since one of the most important problems that confronts the author of a unified course in mathematics, is the proper selection from a wealth of material, it seems unwise to devote twelve pages to spherical trigonometry and deal scantily with analytic geometry.

3. Despite the author's assertion in the preface that "this book is not intended to be a text on 'Practical Mathematics,' in the sense of making use of scientific material and of fundamental notions not already in the possession of the student, or in the sense of making the principles of mathematics secondary to technique," the book has the appearance of a manual on practical mathematics.

On the first reading one is inclined to say that the book has been written from the engineering point of view. But while the author deals elaborately with the practical applications of all of the mathematical material that the student has on hand or acquires throughout the course, he none the less lays emphasis on the proper understanding of mathematical principles and, as regards technique and application, strives to maintain the proper balance. Just how well he has succeeded in doing this it is hard to say.

The book will certainly arouse interest and provoke thought. It shows breadth of thought both in conception and execution. It is impossible to pass judgment, without a fair trial, on its usefulness as a textbook.

The first three chapters of the book deal with scales, rectangular coördinates, variables and functions, and the power function. In our opinion this part of the work is given in too great detail. The author has an admirable way of presenting transformations early in the course. Certain theorems on loci are stated in connection with the graphs of functions and these theorems explain in a simple but satisfactory way translation, rotation, expansion and shear.

In Chapter IV the trigonometric functions are defined for any angle and the treatment of the functions is taken up in connection with the analytic geometry of the circle. While Chapter V, on the ellipse and hyperbola has some connection with Chapter IV, there is a rather abrupt change in passing to the selected topics of algebra in Chapters VI–VIII. After this there is another change in taking up the logarithmic and exponential functions of Chapter IX and the remainder of the work in trigonometry in Chapter X. A better arrangement seems possible. Why can not Chapter IX follow Chapter III and Chapter X follow Chapter IV? This seems more desirable and more natural from the point of view of coherence. The functionality idea would thus be predominant in

the first four chapters and there would be a natural shift through the circular functions to the two following chapters in trigonometry.

A review of the most important topics of secondary school algebra is found in the twenty-four pages of the appendix.

4. To what extent have the authors succeeded in unifying algebra, trigonometry and analytic geometry and what is the process of unification? They have not chosen the way of functionality. The notion of a function first appears on page 58, and while it is constantly in use throughout the book, it is not predominant and does not determine the arrangement of the subject matter.

The part of the book that has been unified in a splendid way is the first seventeen chapters (280 pages). This includes the application of algebra to arithmetic, trigonometry and its applications, and the more elementary part of analytic geometry. The graphical method serves in a most natural way as a connecting link. A characteristic feature of this part of the book is the "timing exercises." Wherever a set of exercises is given for the purpose of developing the students' mechanical ability, a tentative time is specified. This will undoubtedly stimulate interest and challenge the students' mental activity. The next seven chapters are devoted to the conics and their applications. The most important properties of the conics are derived and the subject is enlivened by a splendid collection of practical problems. In connection with the practical problems on elliptic and parabolic arches, there are several photographs of bridges where these arches are used. There are also photographs showing the use of elliptic gears.

The solution of numerical equations, wave motion, laws of growth, polar coördinates, complex numbers, and solid analytic geometry are the subjects considered in the last eight chapters. The two most interesting chapters in this part of the book are those on wave motion and laws of growth. The authors have succeeded splendidly in presenting, in connection with the more formal part of the work, interesting problems that have practical application. Indeed this is a distinctive feature of their book. The problem about the big gun that bombarded Paris and the curve of healing of a wound, give evidence that they are up-to-date. While they have placed much emphasis on the practical application of the subject matter, they have not neglected the more formal part of the work.

There is some doubt as to how far they have succeeded in unifying the later material. The latter part of the book seems to lack the coherence that is characteristic of the first part. Perhaps this is due to the fact that trigonometry forms the body of the first part.

There are many historical notes throughout the book.

The regularity of the type gives it an attractive appearance.

The following tables are bound in with the text: squares, cubes, square roots, cube roots, and reciprocals (from 1 to 100); logarithms; natural functions; radian measure of angles; tables for e^x and e^{-x}; tables of interest functions.

Having examined these four books in detail let us now take a broader view of them.

So far as unification is concerned, they have a common purpose, though Ransom does not give as much attention to coherence as the others do. There are differences however in the subject matter as well as in the way by which unity may be accomplished. There is also a certain difference in the spirit of the books. The chief purpose of McClenon and Rusk seems to be to unify the subject matter with which they are concerned. The other three books breathe a different spirit. They, too, seek coherence but, in addition to this, they agree that a prominent feature of the work of the freshmen, should be the use of interesting and helpful problem material—material that brings together the workaday world and mathematical principles. They have succeeded admirably in collecting such material.

As regards subject matter, there is rather close agreement on the work in trigonometry. Has enough attention been given to trigonometry or has it been cut down to an irreducible minimum? In McClenon there are 73 pages on trigonometry and its application; in Ransom, 76 pages; in Slichter, 90 pages; in Karpinski, Benedict and Calhoun 114 pages. In our opinion the best arrangement of the trigonometry is given by Karpinski, Benedict and Calhoun. Ransom scatters the subject throughout the book in six different parts.

There is disagreement about the topics in college algebra. McClenon and Rusk confine their attention to graphic algebra. Ransom devotes 32 pages to determinants and the solution of numerical equations; Slichter gives 17 pages to permutations and combinations; Karpinski, Benedict and Calhoun give 14 pages to the solution of numerical equations.

Analytic geometry with its applications is given far greater emphasis by Karpinski, Benedict and Calhoun, and Slichter than by the other two authors. Since Slichter and Karpinski lay so much emphasis on the practical application of mathematics, it is rather surprising that they do not give the fundamental ideas of the differential and integral calculus, and their applications to the simplest functions. In Karpinski the work on poles and polars, transformations, polar coördinates, complex numbers, and solid analytic geometry covers about 90 pages. In our opinion it would be more profitable to devote this space to the calculus. Slichter could also find time for the calculus by not going into such elaborate detail in different parts of the book. Many of his directions, specifications and remarks could be left to the teacher.

The tendency to make as wide use as possible, of practical problems in illustrating mathematical principles is a step in the right direction. When the teachers of mathematics know more of its applications and make wider use of them, and when the teachers of allied subjects call into use the mathematics that the student may have in his possession, there will then be a greater recognition of its supreme importance.

It means little to praise or criticize these books without first giving them a fair trial. Whatever their faults, this new arrangement of material should be tested in the class room. We welcome the books as a contribution towards the solution of our problem. They will be of service to every mathematics teacher of college freshmen.

GEORGE W. MULLINS.

Analytic Geometry. By MARIA M. ROBERTS and JULIA T. COLPITTS. New York, Wiley, 1918. 10 + 229 pp. Price $1.60.

The following extracts from the preface serve to indicate the general plan of the book: "This book is the result of several years of experience in teaching mathematics to students of engineering and science. . . . Emphasis has been placed on those portions of analytic geometry in which experience has shown the student of calculus to be most frequently deficient. In this connection, in particular, polar coördinates have received more than usual attention and transcendental and parametric equations considerable space. . . . The material is so arranged that the first ten chapters together with a portion of Chapter XIII include those subjects ordinarily offered to such freshman classes as cover in the first year the three subjects, college algebra, trigonometry and analytic geometry. The addition of Chapter XIV will round out a good course of five hours a week for a semester. The entire book should easily be covered in a three-hour course throughout a year."

Chapter I (17 pages) is entitled Cartesian Coördinates. After the usual introduction, the formulæ for distance between two points, the slope of a line, the point of division, and the area of a triangle, are introduced and applied to numerical examples. Some familiar theorems from plane geometry are introduced for proof by analytic methods.

Chapter II (29 pages) is devoted to Loci. Here algebraic equations only are considered. In deriving the equation of a locus emphasis is placed on the fact that the student must show "why the coördinates of all points off the locus fail to satisfy the equation." Plotting the locus of an equation is considered at length. Most of the examples involve conic sections, although a few higher plane curves are introduced. The outline for the discussion of an equation is very good, particularly the section on extent of the curve. Curves with asymptotes parallel to one of the coördinate axes are introduced. The chapter closes with an interesting section on loci through the intersections of two given loci.

Chapter III (26 pages) is devoted to the straight line. Here the proof for the normal form of the equation, which is made to depend on the intercept form, is not satisfactory since it does not hold when the line passes through the origin. Moreover there seems to be no good reason why p in this equation should not be considered as always positive, instead of positive above the x axis and negative below as suggested by the authors. In fact, in the later chapter on solid analytic geometry, the perpendicular distance from the origin to a plane is always considered positive.

The fourth Chapter (17 pages) introduces polar coördinates. The treatment here is excellent but the subject is one which offers serious difficulties to the student. Since polar coördinates are not used elsewhere in the book, save in the chapter on transformation of coördinates, Chapter IV might well come later, after the student has acquired facility in methods of analytic geometry.

Transformation of coördinates comes in the fifth chapter, very appropriately preceding the chapters on the various conic sections. The treatment here is

brief (10 pages) but adequate. Translation as well as rotation of axes are considered and equations are simplified by each of these transformations. Examples involving the removal of the xy term are limited, however, to cases where a rotation of 45° is called for. The chapter concludes with a section on transformation from rectangular to polar coördinates and vice versa.

Chapters VI (20 pages), VII (10 pages), VIII (12 pages) and IX (14 pages) take up the circle, parabola, ellipse and hyperbola, respectively. In the case of the circle it seems hardly necessary to call $(x - h)^2 + (y - k)^2 = r^2$ the "First standard equation of a circle" and $x^2 + y^2 = r^2$ the "Second standard equation." Similarly the four standard equations of the ellipse and of the hyperbola could well be reduced to two, the others being merely special cases.

The chapter on the parabola begins with a derivation of the rectangular equation of a conic section from the ratio definition. The ratio definition is used also in deriving the equations of the ellipse and of the hyperbola, the familiar properties involving respectively the sum and difference of the focal radii being proved as theorems, not assumed as definitions. This seems on the whole the best method of approach. The treatment here is brief, only the fundamental properties of the various conics being discussed.

Tangents and normals are considered in the tenth chapter of twelve pages. The slope of a tangent is derived by means of the disguised calculus method with h and k substituted for Δx and Δy. This method is applied in particular to the general equation of the second degree, the resulting rule being very convenient in numerical examples. The chapter concludes with a paragraph on the lengths of tangents, normals, subtangents and subnormals and one on the equation of the tangent when the slope is given.

The remaining chapters take up topics of a more advanced nature and are not intended, as the preface indicates, for freshman classes. Poles, polars, diameters and confocal conics are discussed in the eleventh chapter of thirteen pages. The notion of pole and polar is introduced by means of harmonic division. The other topics are treated in the usual way.

A brief chapter (the twelfth) of only six pages follows. This takes up the general equation of the second degree. Methods for the classification of conics are developed and examples involving conics through five given points solved.

Chapter XIII (20 pages) involves transcendental and parametric equations. The usual logarithmic and exponential curves are plotted as well as the graphs of trigonometric and inverse trigonometric functions. Under parametric equations, some of the more important curves considered are the cycloid, the epicycloid, the hypocycloid, the path of a projectile, the witch and the cissoid.

The concluding chapter, numbered fourteen, is one of the best of the book. The material here is well selected and well presented. In the brief space of thirty-three pages one finds practically all the material regarding solid analytic geometry necessary for a first course in the calculus. In addition to the standard proofs and examples concerning the line and plane, the chapter includes discussions of cylindrical surfaces, spherical surfaces, surfaces of revolution, the more

important forms of quadric surfaces and the equations of a space curve. The statement that the polar coördinates of a point in space are ρ, α, β and γ (ρ being the radius vector and α, β and γ the direction angles) is not in accordance with the usual notation for polar coördinates. Moreover the use of four coördinates for a point in some cases and three in others is confusing to the student, even though he is told that the four are not independent.

The problems in the book are almost entirely from the field of pure mathematics, few from engineering or other sources being used. The answers are given for most of the problems, either immediately after the statement of the problem or in a list of answers at the end of the book. Sets of miscellaneous examples inserted from time to time furnish an opportunity for review.

As already indicated in quotations from the preface, the book is conveniently arranged for use either in a course for a term, a semester or a year. The traditional over emphasis on conic sections is relieved by the excellent chapters on polar coördinates, transcendental equations and parametric equations.

CLINTON H. CURRIER.

PROVIDENCE, R. I.,
March 1, 1919.

Lezioni sulla teoria dei gruppi continui finiti di transformazioni. By L. BIANCHI. Pisa, E. Spoerri, 1918. 8vo. 6 + 590 pp. Price 25 lire.

This volume was written for the students of the Italian universities and is a reproduction, with few changes, of the course by the same author which appeared in lithographed form about fifteen years ago. It constitutes the second introductory treatise on continuous groups in the Italian language, an earlier one having been published by G. Vivanti under the title *Teoria dei gruppi di transformazioni*, 1898; translated into French by A. Boulanger, 1904.

The interest of Italian mathematicians in the general field of group theory is reflected by the fact that they have now in their own language at least five books on this subject. Two of these relate to finite groups, viz., the translation into Italian of Netto's *Substitutionentheorie* by G. Battaglini, 1885, and the excellent introduction entitled *Lezioni sulla teoria dei gruppi di sostituzioni*, 1910, by the author of the work now under review. Their other books on group theory are devoted to the groups of analysis and geometry, and include the well-known *Teoria dei gruppi discontinui e delle funzioni automorfe* by G. Fubini, 1908, in addition to the two works mentioned in the preceding paragraph.

L. Bianchi is well known to American mathematicians as the author of good books relating to each of the three great fields of mathematics—algebra, analysis and geometry. His wide range of mathematical knowledge and his extensive experience as a writer have qualified him admirably for the production of a work having such varied contact as a treatise on continuous groups should exhibit. In the present work he has restricted himself largely to a lucid exposition of the theories contained in the first volume of the *Theorie der Transformationsgruppen* by Lie-Engel, 1888–1893.

The book is divided into thirteen chapters. The first of these is devoted to a brief exposition of auxiliary theories relating mainly to systems of differential equations. In chapters II to XI inclusive there is given a very clear exposition of the fundamental concepts of continuous groups, exclusive of the important theory of contact transformations. The last two chapters are devoted to applications. In the former of these, Chapter XII, the author gives the fundamental concepts relating to the applications of the theory of continuous groups in the theory of differential equations. The final chapter is devoted to geometric applications and bears the heading "Applicazioni alla teoria degli spazi pluridimensionali con gruppi continui di movimenti."

Although the present work is considerably more concise than the *Vorlesungen über Continuierliche Gruppen*, 1893, by Lie-Scheffers it is almost equally elementary and leads the student much more rapidly to the fundamental theorems of the subject. The book is unusually free from errors and constitutes a decidedly useful aid for acquiring quickly an insight into the abstract theory of finite continuous groups. While the founder of the theory of continuous groups was a Norwegian he published most of his extensive systematic developments in the German language, and the present work will be especially welcomed by those who desire to acquire a knowledge of this field but do not read German comfortably.

G. A. MILLER.

An Introductory Treatise on Dynamical Astronomy. By H. C. PLUMMER. Cambridge: at the University Press, 1918. Royal 8vo. 19 + 343 pp. Price 18 shillings.

Contents—Chapter I: The law of gravitation, 1–10; II: Introductory propositions, 11–20; III: Motion under a central attraction, 21–32; IV: Expansions in elliptic motion, 33–48; V: Relations between two or more positions in an orbit and the time, 49–64; VI: The orbit in space, 65–72; VII: Conditions for the determination of an elliptic orbit, 73–84; VIII: Determination of an orbit, Method of Gauss, 85–93; IX: Determination of parabolic and circular orbits, 94–102; X: Orbits of double stars, 103–114; XI: Orbits of spectroscopic binaries, 115–128; XII: Dynamical principles, 129–141; XIII: Variation of elements, 142–157; XIV: The disturbing function, 158–176; XV: Absolute perturbations, 177–191; XVI: Secular perturbations, 192–206; XVII: Secular inequalities, Method of Gauss, 207–217; XVIII: Special perturbations, 218–235; XIX: The restricted problem of three bodies, 236–253; XX–XXI: Lunar theory, 254–291; XXII: Precession, nutation and time, 292–311; XXIII: Libration of the moon, 312–322; XXIV: Formulæ of numerical calculation, 323–340; Index, 341–343.

Quotations from the preface: "This book is intended to provide an introduction to those parts of Astronomy which require dynamical treatment. To cover the whole of this wide subject, even in a preliminary way, within the limits of a single volume of moderate size would be manifestly impossible. Thus the treatment of bodies of definite shape and of deformable bodies is entirely excluded, and hence no reference will be found to problems of geodesy or the many aspects of tidal theory. Already the study of stellar motions is bringing the methods of statistical mechanics into use for astronomical purposes, but this development is both too recent and too distinct in its subject-matter to find a place here. . . . Since the main object in view has been to cover a wide extent of ground in a tolerably adequate way rather than to delay over critical details, the absence of mathematical rigour may sometimes be noticed. Very little attention is given to such questions as the convergence of series. It is not to be inferred that these points are unimportant or that the modern astronomer can afford to disregard them. But apart from a few simple cases where the reader will either be able to supply what is necessary for himself, or would not benefit even if a critical discussion were added, such questions are extremely

difficult and have not always found a solution as yet. It is precisely one of the aims of this book to increase the number of those who can appreciate this side of the subject and will contribute to its elucidation."

Mr. Plummer is professor of astronomy in the University of Dublin and Royal Astronomer of Ireland.

Handbook of Mathematics for Engineers. By E. V. HUNTINGTON. With Tables of Weights and Measures by L. A. FISCHER. New York, McGraw-Hill, 1918. 12mo. 5 + 191 pp. Price $1.50.

This volume is a reprint of sections 1 and 2 of the *Mechanical Engineers' Handbook* edited by L. S. Marks (1916). Mr. Fischer's tables occupy pages 70–85. The rest of the work is by Professor Huntington.

The "mathematical tables" (pages 2–69) are: squares of numbers, cubes of numbers, square roots of numbers, cube roots of numbers, three-halves powers of numbers, reciprocals of numbers, circles (areas, segments, etc.), spheres (volumes, segments, etc.), regular polygons, binomial coefficients, common logarithms, degrees and radians, trigonometric functions, exponentials, hyperbolic (Napierian) logarithms, hyperbolic functions, multiples of 0.4343 and 2.3026, residuals and probable errors, compound interest and annuities, and decimal equivalents.

The contents of the rest of the work are as follows: Arithmetic (Numerical computation, logarithms, the slide rule, computing machines, financial arithmetic), 88–98; Geometry and Mensuration (Geometrical theorems, geometrical constructions, lengths and areas of plane figures, surfaces and volumes of solids), 99–111; Algebra (Formal algebra, solution of equations in one unknown quantity, solution of simultaneous equations, determinants, imaginary or complex quantities), 112–127; Trigonometry (Formal trigonometry, solution of plane triangles, solution of spherical triangles, hyperbolic functions), 128–135; Analytic Geometry (The point and the straight line, the circle, the parabola, the ellipse, the hyperbola, the catenary, other useful curves), 136–156; Differential and Integral Calculus (Derivatives and differentials, maxima and minima, expansion in series, indeterminate forms, curvature, table of indefinite integrals, definite integrals, differential equations), 157–172; Graphical representation of functions (Equations involving two variables, equations involving three variables, equations involving four variables), 173–184; Vector Analysis, 185–186; Index, 187–191.

The volume contains a great amount of useful and interesting information admirably edited.

The American Society of Mechanical Engineers, New York. *The Weights and Measures of Latin America*[1] by F. A. HALSEY. New York, 1918. 8vo. 34 pp.

This is a report based on the replies received after distributing five hundred copies of a questionnaire throughout South and Central America and the West Indies. Four of the six questions of the questionnaire were as follows:

1. What are the units of weight and measure commonly used with relation to the buying and selling at retail of the following products?—Groceries, fruits, milk, butter and cheese, other farm products, hardware, fish, meat, flour, tea and coffee, dry goods, fuel, tobacco, miscellaneous.

2. What are the units of measure commonly used with relation to buying and selling articles of clothing, as follows?—Ready made clothing, hats, collars, underwear and hosiery, shoes, gloves, corsets, miscellaneous.

3. What are the units of measure commonly used with relation to the sale of lands and filing of paper and deeds as follows?—In the farming districts, in the smaller towns, in the cities.

6. What are the units of weight and measure commonly used with relation to transportation tariffs?—Railway tariff for passengers and freight (load and distance), loads and rates for city transportation, loads and rates for transportation by muleback across the mountains, railway track gages and length of lines, railway equipment (units used in the construction and repairing of locomotives, cars, etc.).

[1] Paper presented at the annual meeting of The American Society of Mechanical Engineers, December, 1918.

The inquiry sets forth the extent to which the metric system has been adopted by Latin America.

Cours d'analyse mathématique. Par E. GOURSAT. Troisième édition revue et augmentée. Tome 2, Paris, 1918. 8vo. 2 + 672 pp. Price (unbound) 36 francs.

Quotation from the preface: "Cette nouvelle édition ne diffère de la précédente que par quelques additions, dont la plus importante est relative à une proposition célèbre de M. Picard. Ce théorème a fait l'objet d'un grand nombre de travaux, qui ont conduit à une démonstration presque élémentaire, ne faisant appel qu'à des inégalités classiques de la théorie des séries entières. Il m'a semblé qu'une démonstration de cette nature avait sa place marquée dans un Cours d'Analyse."

The third edition of tome 2 contains 24 pages more than the second edition.

NOTES.

In *A Calendar of Leading Experiments* by W. S. FRANKLIN and B. MACNUTT (South Bethlehem, Pa., Franklin, MacNutt, and Charles, 1918), Part I (pages 1–68) is entitled, "Mechanics," and Part VI (pages 163–205), "The dynamics of wave-motion."

In *The California Alumni Fortnightly*, volume 12, page 131, May 3, 1919, there is a portrait of Professor FRANK MORLEY, who is to teach in the Summer School of the University of California.

The Carnegie Institution of Washington has published the first volume of Professor L. E. DICKSON's monumental *History of the Theory of Numbers* (Paper, $7.50; cloth, $8.00). The second volume is in the press.

An Italian translation of J. W. Young's *Lectures on Fundamental Concepts of Algebra and Geometry* has been recently published by Luigi Pierro, Naples.[1] The translated text has neither been enlarged nor diminished in the new edition. The translator has, however, added a large number of explanatory and bibliographical notes.

Wiley has just published *Lectures on Ten British Physicists* by the late ALEXANDER MACFARLANE. It is a companion volume to his *Lectures on Ten British Mathematicians* published in 1916 and it is No. 20 in the series of "Mathematical Monographs." The physicists considered are: James Clerk Maxwell (1831–1879); William John Macquern Rankine (1820–1872); Peter Guthrie Tait (1831–1901); Sir William Thomson, First Lord Kelvin (1829–1907); Charles Babbage (1791–1871); William Whewell (1794–1866); Sir George Gabriel Stokes (1819–1903); Sir George Biddell Airy (1801–1892); John Couch Adams (1819–1892); Sir John Frederick William Herschel (1792–1871).

In *The American Year Book . . . 1918* (New York, Appleton, 1919) are articles on "Mathematics" (pages 615–617) by G. A. Miller, and on "Astronomy"

[1] *I Concetti fondamentali dell'Algebra e della Geometria,* Versione e Note di Domenico Mercogliano, Con Prefazone di Gino Loria 1919. 418 pp. Price 8 lire.

(pages 617–622) by R. S. Dugan. The five divisions of the article on mathematics are entitled: 1. History of Mathematics; 2. Effects of the War on Mathematical Advances; 3. Advances in Pure Mathematics; 4. Teaching of Mathematics; 5. Personal Notes. It is stated that Professor Florian Cajori "enjoys the unique distinction of being the first regular university professor of this history [of mathematics] in the world." There is no mention of America's mathematical contributions to ballistics and other branches of applied mathematics. Under Personal Notes there are references to one foreigner, G. Cantor, "founder of the theory of aggregates," who died January 6, and to eight Americans.

Encyclopædia of Religion and Ethics. Edited by JAMES HASTINGS with the assistance of J. A. Selbie and L. H. Gray. Edinburgh, Clark, Volume 9, 1917; Volume 10, 1919. These volumes include the following articles of interest to the mathematician:

"Numbers"—Introductory by T. Davidson, 406–407; Aryan by A. B. Keith, 407–413; Semitic by W. Cruickshank, 413–417.

"Pascal" by W. F. Cobb, 652–658; one of the three sections of the biography is entitled "His mathematical aptitude."

"Greek Philosophy" by Paul Shorey, 859–865.

"Plato and Platonism" by Henry Jackson, 54–61.

"Points of the Compass" by T. D. Atkinson, 73–88.

"Pythagoras and Pythagoreanism" by John Burnet, 520–530.

A New English Dictionary on Historical Principles (Oxford, Clarendon Press) —the greatest dictionary, in any language, ever published—is nearing completion. The part issued in March, 1918, contains discussions of the following words of interest to the mathematician: Supplement, surd, surdesolid and sursolid, surface, surveying and surveyor, suversed, and swan-pan (*also* souan-, shwan-, swam-, suan-). The Chinese word Soroban, which occurs frequently in English writings, and is another equivalent of swan-pan, is not given in the '*N.E.D.*'

The earliest quotations given in connection with some of these words as mathematical terms are as follows: 1570, Billingsley's *Euclid*, Book 1, Theorem 32, "In every parallelograme, the supplementes of those parallelogrammes which are about the diameter, are equall the one to the other"—1551, Recorde's *Path. Knowl.* II. Pref., "Quantitiees partly rationall, and partly surde." [Surd is derived from the Latin word *surdus* meaning deaf. "The mathematical sense 'irrational' arises from Latin *surdus* being used to render Greek ἄλογος (Euclid, bk. 10, Def.), apparently through the medium of Arabic *açamm*, deaf."]—1557, Recorde's *Whetst.* G iiib, "That root is a Sursolide roote, that yieldeth a Sursolide nomber"—[Suversed = supplement + versed; suversed sine: the versed sine of the supplement.[1]] 1827, Airy in *Encycl. Metrop.* (1845) I, 674, "The versed sine

[1] That is, suvers $x = 1 + \cos x$. The term suversed was used before 1827; for example in: (1) J. de Mendoza Rios, *A complete Collection of Tables for Navigation and Nautical Astronomy*, London, 1805; (2) W. Lax, *Tables to be used with the Nautical Almanac for finding Latitude and Longitude at Sea*, London, 1821; (3) E. Riddle, *Treatise on Navigation and Nautical Astronomy . . . with all the Tables requisite . . .* London, 1824.

of one is the suversed sine of the other"—1736 tr. *Du Halde's Hist. China*, III, 70," "In casting up Accounts they [the Chinese] make use of an Instrument called Souan pan." [The term is derived from the Chinese and means, literally, reckoning board.]

ARTICLES IN CURRENT PERIODICALS.

AMERICAN JOURNAL OF MATHEMATICS, volume 41, no. 1, January, 1919: "Groups generated by two operators whose relative transforms are equal to each other" by G. A. Miller, 1–4; "A classification of general (2, 3) point correspondences between two planes" by T. R. Hollcroft, 5–24; "The classification of plane involutions of order (3)" by Anna M. Howe, 25–48; "On surfaces containing a system of cubics that do not constitute a pencil" by C. H. Sisam, 49–59; "An isoperimetric problem with variable end-points" by A. S. Merrill, 60–78—No. 2, April: "Asymptotic satellites near the straight-line equilibrium points in the problem of three bodies" by D. Buchanan, 79–110; "Concerning the invariant theory of involutions of conics" by W. Sensenig, 111–122; "Note on seminvariants of partial differential equations" by A. L. Nelson, 123–132; "On a method for determining the non-stationary state of heat in an ellipsoid" by B. Datta, 133–142; "Nilpotent algebras generated by two units, i and j, such that i^2 is not an independent unit" by G. W. Smith, 143–164.

ANNAES SCIENTIFICOS DA ACADEMIA POLYTECHNICA DO PORTO, volume 11, 1916, no. 1: "Sur différents procédés d'approximation" by A. Aubry, 5–35; "Timorenses de Okussi e Ambeno" by A. A. Mendes Corrêa, 36–51; "Palavras proferidas na sessão do lançamento da prime ra pedra da Escola de Pharmacia da Universidade do Porto no dia 1 de fevereiro de 1915," 52–57; "Quelques considérations sur les systèmes de formes linéaires" by Miss Velleda Gradara, 58–64—No. 2: "Recherches des involutions de genres zéro, bigenre un, appartenant à une surface de genres un" by L. Godeaux, 65–78; "Discurso proferido na sessão solemne do lançamento da primeira pedra a construcção do edificio da Escola de Farmacia do Porto" by N. F. Dias Salgueiro, 79–88; "Sôbre a correlação de certos índices mandibulares com o índice cefálico" by A. Pires de Lima, 89–103; "Essai d'une théorie analytique des lignes non-euclidiennes" (continuation) by G. Pirondini, 104–124; "L. Orlando" by G. T[eixeira], 125–126; Review by A. Freire de Andrade of L. F. Navarro's *Cristalografia geométrica elemental* (Madrid, 1915), 127–128—No. 3: "Sur l'aire d'un segment de courbe convexe" by E. Turrière, 129–140; "Essai d'une théorie analytique des lignes non-euclidiennes" (continued), by G. Pirondini, 141–146; "Notas sobre Vernier" by A. Cardoso Pereira, 147–154; Reviews by Gomes Teixeira, of Lecornu's *Cours de Mécaniques* (tomes 1–2, Paris, 1914–1915), Richardson and Landis's *Fundamental Conceptions of Modern Mathematics* (Chicago, 1916), Archibald's *Euclid's Book on Divisions of Figures* (Cambridge, 1915), and L. O. de Toledo's *Elementos de Aritmética Universal*, tome 2 (Madrid, 1916), 185–188; Review by Mendes Corrêa of Frassetto's *Diagnosi e valutazione numerica delle curve in antropometrica ed in biometrica* (Rome, 1916), 189–190—No. 4: "Extrait d'une lettre adressée à F. Gomes Teixeira" by E. Turrière, 193–195; "Sur les formes isoclines et le problème Diophantique qui en découle" by V. Souza Brandão, 196–223—Volume 12, No. 1, 1917: "Sur l'aire d'une courbe plane générale" by E. Turrière, 5–12; "Sur une intégrale définie dont l'élément est une exponentielle de degré 4" by P. Appell, 12–13; "Sobre a construcção das tangentes á cissoide obliqua que passam por um ponto exterior á curva'' by F. Gomes Teixeira, 14–17; "L'équation tangentielle polaire des courbes de Cesàro" by L. Braude, 18–26; Reviews by G. T[eixeira] of Rey Pastor's *Fundamentos de la geometria proyetiva superior* (Madrid, 1916) and *Introduccion á la matematica superior* (Madrid, 1916), P. Burgatti's *Lezioni di meccanica razionale* (Bologna), Halphen's *Oeuvres* (tome 1, Paris, 1916), Bôcher's *Leçons sur les méthodes de Sturm* (Paris, 1917), and Boutroux's *Les principes de l'analyse mathématique* (Paris, 1914), 53–59—No. 2, 1918: "Los problemas de la mecánica" by E. Terradas, 94–125; "Rapport de M. Appell sur les travaux de M. F. Gomes Teixeira, 126–128, [Reprinted from *Comptes rendus de l'académie des sciences de Paris*, 1917, tome 165, p. 907; Quotation: "En dressant un catalogue raisonné de ces courbes, en donnant leur histoire dans un important ouvrage, M. F. Gomes Teixeira a rendu à la science un grand service, que la Commission propose de reconnaître en lui décernant le prix Binoux"]—No. 4, 1918: "Congresso de sevilha celebrado pela Associação Espanhola para o progresso das sciencias (6–11 de Maio de 1917)" by A. F. de Lacerda, "Sciências matemáticas" 201–204; "Sur la réprésentation géométrique de la torsion d'une courbe gauche," 218–224.

ANNALS OF MATHEMATICS, 2nd series, volume 20, no. 3, March, 1919: "On quaternions and their generalization and the history of the eight square theorem" by L. E. Dickson, 155–171; "Non-symmetric kernels of positive type" by Caroline E. Seely, 172–176; "Elementary properties of the Stieltjes integral" by H. E. Bray, 177–186; "A kinematical property of ruled surfaces" by J. K. Whittemore, 187–190; "Systems of linear inequalities" by L. L. Dines, 191–199; "On the shortest line between two points in non-Euclidean geometry" by T. H. Gronwall, 200–201; "The generalized gamma functions" by E. L. Post, 202–217; "On the most general plane closed point-set through which it is possible to pass a simple continuous arc" by R. L. Moore and J. R. Kline, 218–223; "Repeated integrals" by D. C. Gillespie, 224–228.

ATHENAEUM, London, 1919, April 18: "Dreams and facts," Part I by Bertrand Russell, 198–199; Review of Russell's *Introduction to Mathematical Philosophy* (London, 1919), 209–210—April 25: "Dreams and facts," Part II by B. Russell, 232–233.

BIOMETRIKA, Cambridge, Eng., volume 12, parts 1–2, November, 1918: "On the standard deviations of adjusted and interpolated values of an observed *polynomial function* and its constants and the guidance they give towards a proper choice of the distribution of observations" by Kirstine Smith, 1–85; "On the product-moments of various orders of the normal correlation surface of two variates" by K. Pearson and A. W. Young, 86–92; "The correlation coefficient of a polychroic table" by A. Ritchie-Scott, 93–133; "On a formula for the product-moment coefficient of any order of a normal frequency distribution in any number of variables" by L. Isserlis, 134–139; "On the mathematical expectation of the moments of frequency distributions" by A. A. Tchouproff, 140–169; "Sur les moments de la fonction de corrélation normale de n variables" by S. Bergström, 177–183; "Formulæ for determining the mean values of products of derivatives of mixed moment coefficients in two to eight variables in samples taken from a limited population" by L. Isserlis, 183–184.

COLUMBIA UNIVERSITY QUARTERLY, Volume 21, No. 2, April 1919: "The new wisdom" by C. J. Keyser, 118–124. [First paragraph: "It is evident that to meet the great demands of the coming time men and women must needs have a spiritual equipment—intelligence, imagination, sympathy, understanding—far surpassing that which has hitherto been deemed sufficient. For in the time to come men and women cannot be, as hitherto, mere dwellers in a province or mere members of a commonwealth, a state, or a nation. Whether they will or no, they are destined to be, in respect of their interests and obligations, citizens of the world. As competent citizens of the world they will have to be either competent leaders in the affairs of world citizenship or—what is not less important—competent judges of such leadership. Provincial wisdom, however precious or fine, cannot longer suffice. What is demanded is a certain large intelligence—a certain wisdom, as we may call it—about the world."]

EDUCATION, Boston, volume 39, no. 8, April, 1919: "A review book in mathematics" by R. R. Goff, 471–472.

JOURNAL OF EDUCATION, London, volume 51, February 1, 1919: "Mathematical problem papers" by C. Davison, 90–91; "Mathematical Association," 130.

JOURNAL OF THE WASHINGTON ACADEMY OF SCIENCES, volume 9, no. 3, February 4, 1919: "A contribution to quantitative epidemiology" by A. J. Lotka, 73–77.

MATEMATISK TIDSSKRIFT, Copenhagen, 1919, no. 1, February, *A*: "H. G. Zeuthen" (Portrait frontispiece), 1–2 [In honor of the 'Nestor of Danish mathematicians' whose eightieth birthday occurred on February 15, 1919. He was editor of the *Tidsskrift*, 1871–1889]; "Trekantens Vinkelsum" by J. Hjelmslev, 3–11; "De matematiske Opgaver ved Mellemskoleeksamen og Realeksamen. En kritisk Vurdering" by F. Friss-Petersen, 11–25; Review of *Tidsskrift för elementär Matematik, Fysik och Kemi*, Stockholm, 1917–18, 28–30; "Danske Eksamensopgaver," 30–37—*B*: "Til Tidsskriftets Laesere og Medarbejdere" by H. Bohr and T. Bonnesen, 1–2; "Det ojeblikkelige Drejningspunkt" by J. Hjelmslev, 2–14; "Om den Hadamard'ske 'Hulsætning'" by H. Bohr, 15–21; Review by T. B[onnesen] of Hjelmslev's Lærebog i Geometri til Brug ved den polytekniske Laereanstalt (København, 1918), 21–26; "Del af polyteknisk Eksamen, 1918– Deskriptiv Geometri" 26–28.

MATHEMATICS TEACHER, volume 11, no. 3, March, 1919: "Introductory course in mathematics" by D. E. Smith, 105–114; "First-year algebra, as developed in the academic high school, New Britain, Conn." by R. R. Goff, 115–117; "Indeterminate forms in trigonometry" by M. O. Tripp, 118–120; "The Courtis tests in arithmetic" by P. A. Boyer, 121–132; "Report of the committee to recommend a suitable program in mathematics for Junior High Schools" (H. D. Gaylord, chairman), 133–140; "Platform of the allied associations of public school teachers of Baltimore," 141; "Book Reviews" and "Notes and News" 142–144.

MIND, London, volume 43, July, 1918: "A general notation for the logic of relations" by C. D. Broad, 184–303; "A proof that any aggregate can be well-ordered" by P. E. B. Jourdain, 386–388. [First sentence: "That any aggregate can be well-ordered was stated by Georg Cantor, but he never succeeded in proving this theorem (*cf.* Cantor's *Contributions to the Founding of the Theory of Transfinite Numbers*, English translation, Chicago and London, 1915, pp. 60, 62–63, 66, 90. 109, 204–206)."]—No. 108, October: "On the relation between induction and probability (Part I)" by C. D. Broad, 389–404; "Note" by C. D. Broad, 508. [The "note": "I see, on looking through my paper in *Mind*, N.S., No. 107, that, although I at first speak of 'Whitehead and Russell' I later generally refer to pieces of notation contained in *Principia Mathematica* as 'Russell's.' I did not mean by this to ascribe them to Mr. Russell rather than to Dr. Whitehead. I have no idea which of the authors is responsible for any given part of the book, and I only used 'Russell' as an abbreviation for 'Russell and Whitehead.' I should be sorry indeed to appear unfair to Dr. Whitehead, and my only excuse is the extreme difficulty of putting a phrase like 'Russell and Whitehead' into the possessive in English. Perhaps Dr. Whitehead will provide me with a suitable notation for this purpose, and Mr. Russell will guarantee it to be 'ethically neutral.'"] Volume 44, January, 1919: "Notes on Zeno's arguments on motion" by P. E. B. Jourdain, 123–134; [Quotation: "The following notes have to do with two points. The first is to call attention to an argument used by Mr. R. A. P. Rogers; [In his paper 'On transfinite numbers, and some problems relating to the structure of actual space and time,' *Hermathena*, Vol. 15, 1910, pp. 397–415.] the second is to bring out the force of some remarks attributed to the shade of Zeno on pages 52–55 of the number of *Mind* for January, 1916,[1] and which do not seem to have been expressed clearly enough."] "Note on C. D. Broad's article in the July *Mind* by Bertrand Russell, 124. [The "note": "Mr. Broad's very interesting article . . . attributes to me (for what reason I cannot guess) a number of notations employed in *Principia Mathematica*. As far as my memory serves me, all these were invented by Dr. Whitehead, who, in fact, is responsible for most of the notation in that work. My original notation, before he came to my assistance, may be found in Peano's *Revue de Mathématiques*, vols. 7 and 8."]

NATIONAL REVIEW, London, volume 72, November, 1918: "The position of mathematics" by C. H. P. Mayo; [Reprinted in *Educational Review*, volume 57, March, 1919, pp. 194–204.].

NATURE, volume 103, March 13, 1919: Review by H. S. A. of Bryant's Galileo (London, 1918), 23; "Graphic methods in nautical astronomy" by A. Hutchinson, 25; [Quotations: "In the issue of *Nature* published on October 24 last (vol. 102, p. 155) there appeared an account of an ingenious chart devised by Mr. G. W. Littlehales, of the United States Hydrographic Department, for dealing rapidly with certain problems in nautical astronomy which involve the solution of a spherical triangle when the three sides, or the two sides and the included angle are known. The article is entitled 'A New Graphic Method in Nautical Astronomy,' but it would appear that the idea has been familiar in France for more than five-and-twenty years. The possibility of constructing a chart like that made by Mr. G. W. Littlehales was demonstrated by Maurice d'Ocagne so long ago as 1891 in his work *Nomographie: les calculs usuels effectués au moyen des abaques*, p. 84 and an abacus devised by him on these lines was described in W. Dyck's *Katalog mathematischer und mathematisch-physikalischer Modelle, Apparate und Instrumente*, published in 1892, p. 163. A figure of the chart can be found in a paper by d'Ocagne which appeared in *Journal de l'Ecole Polytechnique* (second series, 4th cahier, 1898, p. 224), and also in his *Traité de Nomographie*, 1899, p. 328. In a modified form the chart was employed by E. Collignon in 1898 (See his 'Note sur la détermination de l'heure du passage du soleil dans un plan vertical,' *Journal de l'Ecole Polytechnique, loc. cit.*, pp. 123–135. . . . The particular cases in the solution of spherical triangles it [the chart] is designed to deal with, frequently occur in the reduction of crystal measurements, and the use of the chart can be confidently recommended to crystallographers"]; "Professor E. C. Pickering," 28–29; "Roger Bacon (1214–94)" by C. Singer, 35–36; [Quotations: ". . . Roger Bacon, great as are his titles to remembrance, was neither the inventor nor introducer of the mariner's compass. . . . His work on this subject [optics] was a text-book for the next two centuries. He saw the importance of lenses and concave mirrors, and showed a remarkable grasp of mathematical optics. He described a system which is equivalent to a two-lens apparatus, and there is trustworthy evidence that he actually used a compound system of lenses equivalent to a telescope. . . . Astronomy was Bacon's perpetual interest. He spent the best part of twenty years in the construction of astronomical tables. His letter to the Pope, in favour of the correction

[1] "The flying arrow: an anachronism," pp. 42–45.

of the calendar, though unsuccessful in his own days, was borrowed and reborrowed, and finally, at third-hand, produced the Gregorian correction. . . . Suggestions described by him include the automatic propulsion of vehicles and vessels. He records also the working out of a plan for a flying-machine. . . . His insistence on the supreme value of mathematics as a foundation for education recalls the attitude of Plato. It was an insistence that the method of thought was more important than its content.

"Summed up, his legacy to thought may be regarded as accuracy of method, criticism of authority, and reliance on experiment—the pillars of modern science. The memory of such a man is surely worthy of national recognition."]—March 20: "Graphical methods in nautical astronomy" by H. B. Goodwin, 44; [Quotations: "As the author of the article in *Nature* of October 24, 1918, in which the diagram referred to by Dr. Hutchinson last week was first brought to the notice of your readers, may I be permitted to supplement the information as to previous efforts in the same direction? . . . M. d'Ocagne . . ., so far as appears at present, is clearly entitled to the credit claimed for him as first in the field. . . . The share of Mr. Littlehales, however, is marked by two features of interest:—(1) That he seems to have been the first to prepare and publish the diagram in a form that promises to be useful in the navigation of air and ocean, and (2) that the simplicity of treatment which deduced the principle and graduation of the chart directly from a formula of spherical trigonometry renders the theory of the matter intelligible to many nautical persons to whom the mysteries of 'Nomographie' are as a sealed book."] "A proof that any aggregate can be well-ordered" by P. E. B. Jourdain, 45; [Quotation: "All the critics of my method sketched or described in my two letters to *Nature* (vol. 101, pp. 84 and 304, 1918), in my two notes in *Comptes rendus* (vol. 166, pp. 520–523 and 984–986, 1918), in *Mind* for July, 1918, and in *Science Progress* for October, 1918, wish to see a certain particular case solved in detail. Although this case does not throw so much light on the problem as the equally simple method of dealing with the general case, which I happen to have discovered long before I applied it to special cases, I here give the treatment of the particular case referred to."] "Ludvig Sylow" by G. B. M[athews], 49.

NYT TIDSSKRIFT FOR MATEMATIK, volume 29, no. 3, January, 1919; *A*: "Några aritmetiska satser" by F. de Brun, 49–54; "Om tal som tallägen" by A. Arwin, 54–63; Review by N. E. Nørlund of A. F. Andersen, H. Bohr and J. Mollerup's *Nyere Undersøgelser over Integralregningen* (København, 1917), 64–68; Review by C. Hansen of E. Landau's *Einführung in die . . . Theorie der algebraischen Zahlen und der Ideale* (Leipzig, 1918), 68–73; "Studentereksamen i Maj 1918—For den matematisk-naturvidenskabelige Linie," 73–75; "Kronik," 77–79—*B*: "Einige Sätze über die ganzen, rationalen Funktionen" by T. Nagel, 53–62; "Skoleembedseksamen Juni 1918," 62–65; "Løninger" by N. Nielsen, 65–68.

PROCEEDINGS OF THE AMERICAN ACADEMY OF ARTS AND SCIENCES, Boston, Volume 54, No. 5, April, 1919: "A new geometrical model for the orthogonal projection of the cosines and sines of complex angles" by A. E. Kennelly, 369–378, 4 plates. [Geometrical constructions for producing plane-vector representations of $\cosh(\theta_1 + i\theta_2)$, $\sinh(\theta_1 + i\theta_2)$, and hence also $\cos(\theta + i\theta_2)$, $\sin(\theta_1 + i\theta_2)$, where $(\theta_1 + i\theta_2)$ is a complex argument, or a complex 'angle,' have been available for some time.[1] The constructions involve a rectangular hyperbola and an associated circle, in one and the same plane, which is the plane of the drawing. By making certain projections in this plane, followed by a rotation through a quadrant, a plane-vector is produced from the origin, corresponding to the complex hyperbolic or circular sine or cosine required. This process is open to the objection that it is somewhat forced and artificial, lacking the simple projective property that a sine or cosine of any real angle possesses in either circular or hyperbolic trigonometry.

More recently, a method of deriving the hyperbolic cosine or sine of a complex angle has been obtained,[2] which enabled the new three-dimensional model here described to be prepared. In this model, it will be seen that the cosine or sine of a complex angle, either hyperbolic or circular, can be produced, by two successive orthogonal projections on to the XY plane, one projection being made from a rectangular hyperbola, and the other projection being made from a particular circle definitely selected among a theoretically infinite number of such circles," concentric but not coplanar.]

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[1] "Two elementary constructions in hyperbolic trigonometry" by A. E. Kennelly, *Annals of Mathematics*, 2d series, Vol. 5, pp. 181–184, July, 1904; reproduced in A. E. Kennelly, *Tables of complex hyperbolic and circular functions*, Harvard University Press, 1914, pp. 165–168.

[2] A. E. Kennelly, *Artificial Electric Lines*, New York, 1917, pp. 120–121.

PROCEEDINGS OF THE NATIONAL ACADEMY OF SCIENCES OF THE UNITED STATES OF AMERICA, volume 5, no. 3, March, 1919: "Tables of the zonal spherical harmonic of the second kind $Q_1(z)$ and $Q_{11}(z)$" by A. G. Webster and W. Fisher, 79–82.—No. 4, April: "On the real folds of Abelian varieties" by S. Lefschetz, 103–106; "Covariants of binary modular groups" by O. E. Glenn, 107–110; "The general solution of the indeterminate equation: $Ax + By + Cz + \ldots = r$" by D. N. Lehmer, 111–114.

RECORD OF THE AMERICAN INSTITUTE OF ACTUARIES, Chicago, volume 7, June, 1918: Review by E. B. Escott of Running's *Empirical Formulæ* (New York, 1917), 55–60.

SCHOOL SCIENCE AND MATHEMATICS, volume 19, no. 4, April, 1919: "The case method of teaching mathematics" by G. A. Miller, 344–349; "On the relations of mathematics to commerce" by R. E. Moritz, 350–357; "Plane geometry for the ninth and tenth grades" by R. R. Goff, 357–358; "Notes on 'Bolshevik Multiplications,'" 359–361; "Problems and solutions," 372–376.

SCIENCE, new series, Volume 44, March 14,1 919: "Cross-section lines on blackboards and their illumination" by P. F. Gaehr, 265. ["Those who wish cross-section rulings on blackboards temporarily, thus leaving the board free for other work after the curve-plotting is finished, can do so by a simple device. On a sheet of white paper make a ruling of lines, 2 cm. apart, the whole grid 16 x 24 cm., and the lines not quite one mm. thick. Take a photograph of this, making the camera image the size of a lantern-slide. Mount the negative in a lantern, projecting the image on the blackboard. A lantern equipped with a 400-watt Mazda lamp will make the lines sufficiently visible for plotting even in a well-lighted room. The lines are erased by turning off the lamp."]—May 9: "Apropos of the proposed historical science section" [of the American Association for the Advancement of Science] by G. A. Miller, 447–448.

SCIENTIA, Bologna, volume 25, no. 3, March, 1919: "The place of Tycho Brahe in the history of astronomy" by J. L. E. Dreyer, 177–185, (French translation, supplément, 39–47); Review by P. E. B. Jourdain of Forsyth's *Theory of Functions of a complex variable*, Third Edition (Cambridge, 1918), 232–233.

SIGMA XI QUARTERLY, volume 6, no. 3, September, 1918: "In memoriam, Henry Shaler Williams (1847–1918)" (Portrait), 53–74. [Professor Williams was the founder of the Sigma Xi Society, and was elected the first president of the first chapter which was established at Cornell University in 1887.]

TEXAS MATHEMATICS TEACHERS' BULLETIN, volume 4, no. 2, February, 1919: "The State Teachers' Association" by J. W. Calhoun, 3; "A neglected opportunity" by H. Y. Benedict, 4–7; "The mathematics club in the high school" by C. E. LaMaster, 8–12; "Military applications of mathematics" by E. J. Oglesby, 13–15; "Factoring in first year algebra" by Mattie R. Watson, 16–18; "Geometry notebooks" by Mrs. Helen Bolton, 19–21; "Combining arithmetic and algebra in the high school" by L. V. Stockard, 22–24; "Can I interest all the pupils in geometry" by Mildred Watkins, 25–28; "The straight edge," 29.

TRANSACTIONS OF THE ROYAL SOCIETY OF CANADA, series 3, volume 12, December, 1918, section 3: "Rational plane anharmonic cubics" by A. M. Harding, 185–195.

AMERICAN DOCTORAL DISSERTATIONS.

L. C. COX, *The finite groups of birational transformations of a net of cubics.* [Reprinted from *American Journal of Mathematics*, volume 39, 1917.] Pp. 59–74. (Cornell, 1915.)

J. V. DE PORTA, *Irrational involutions on algebraic curves.* [Reprinted from *American Journal of Mathematics*, volume 40, 1918.] Pp. 47–68. (Cornell, 1916.)

T. R. HOLLCROFT, *A classification of general* (2, 3) *point correspondences between two planes.* [Reprinted from *American Journal of Mathematics*, vol. 41, 1919.] Pp. 5–24. (Cornell, 1917.)

ANNA M. HOWE, *The classification of plane involutions of order* (3). [Reprinted from *American Journal of Mathematics*, volume 41, 1919.] Pp. 25–48. (Cornell, 1917.)

J. H. MINNICK, *An investigation of certain abilities fundamental to the study of geometry.* Lancaster, Pa., New Era, 1918. Sm. 4to. 8 + 108 pages. (University of Pennsylvania, 1918.)

UNDERGRADUATE MATHEMATICAL CLUBS.

EDITED BY U. G. MITCHELL, University of Kansas, Lawrence, Kan.

CLUB ACTIVITIES.

MATHEMATICS CLUB OF COLUMBIA UNIVERSITY, New York, N. Y. [*1918*, 227–228].

Officers 1918–19: President, Israel Koral '19; secretary, Charles P. Davis '19; faculty adviser, Professor Lewis P. Siceloff.

Programs of meetings for the current semester are as follows:

February 24, 1919: "Groups of operations" by Professor Cassius J. Keyser.

March 10: "Philosophy of mathematics"[1] by Charles P. Davis '19.

March 24: A visit to the mathematical museum, conducted by Professor James Maclay.

April 7: "Projectiles" by August B. Kinzel '20.

THE MATHEMATICAL CLUB, Harvard University, Cambridge, Mass. [*1918*, 186–7, 449–50].

At the first meeting of the current academic year, held in the Common Room of Conant Hall, February 12, Professor William F. Osgood spoke on "Professor Bôcher's Scientific Start in Life." The following is an extract from the minutes of the meeting.

"Professor Bôcher's student days lay in the years just following the establishment of the principal courses in the Department of Mathematics, as we know them today. Instruction in the Calculus at Harvard, it is true, goes back at least to the middle of the eighteenth century, as is clear from the Commencement programmes, and the first catalogues which print the titles of the courses offered show that formal instruction in the subject was given at least as early as 1830. In the years from 1841 to 1846 Benjamin Peirce published his *Curves and Functions*, which for more than three decades served as a text-book in analytic geometry and the calculus.

"With the coming of Professor Byerly to Harvard in 1876, new and more efficient methods of undergraduate instruction were brought into the Department. The solving of problems by the student as a part of the work of each day's assignment is due to him. He found an ardent and efficient supporter in Professor Benjamin Osgood Peirce, who came into the Department in 1881, and these men gave to Mathematics 2 and 5 [the first and the second course in the Calculus] essentially the character they have today. Both men were interested primarily in the applications of the calculus, and the problems they used bear witness to the fact that they believed the calculus should be taught in its relation to physics. It has been so taught at Harvard ever since.

"Mathematics 4 [Mechanics] was Professor B. O. Peirce's work. On the other hand, Professor Byerly created Mathematics 3 [Modern Geometry]. Advanced courses in geometry had been given in the seventies by Professor James Mills Peirce, who was a pupil of Chasles in Paris in the fifties. But he assigned few problems, and the student found the presentation abstract and difficult to understand. Later, Professor Bôcher and Professor Bouton made useful contributions to the subject matter of the course. But its plan and organization had been completed before Bôcher elected it as a student.

"Two further courses which were to be determinative for Professor Bôcher's scientific work in life were Mathematics 10 [Potential Functions and Developments into Series] and 13 [Theory of Functions of a Complex Variable]. The first of these was created jointly by Professor Byerly and Professor B. O. Peirce, and it took a powerful hold on Bôcher while he was still an undergraduate. Indeed, his thesis for Final Honors in Mathematics—he received Highest Final Honors at graduation—was in this field. It was entitled: 'Three Systems of Parabolic Coördinates.'

[1] Cf. Shaw, J. B., *Lectures on the Philosophy of Mathematics*, Chicago, 1918.

"The other course, Mathematics 13, was given by Mr. (at that time not yet Dr.) Frank Nelson Cole, who had just returned from Germany and was aglow with the enthusiasm which Felix Klein inspired in his students. Cole was not the first to give a formal course of lectures at Harvard on the theory of functions of a complex variable, Professor James Mills Peirce having lectured on this subject in the seventies. That presentation was, however, solely from the Cauchy standpoint, being founded on the treatise of Briot et Bouquet, *Fonctions elliptiques*. Cole brought home with him the geometric treatment which Klein had given in his noted Leipzig lectures of the winter of 1881–82. Cole also gave a course in Modern Higher Algebra, with its applications to geometry. The enthusiasm which he felt for his subject was contagious. Interesting as were the other courses I have mentioned, they stood as the Old over against the New, and of the latter Cole was the apostle. The students felt that he had seen a great light. Nearly all the members of the Department,—Professor J. M. Peirce, Dr. B. O. Peirce, and, I think, Professor Byerly,—attended his lectures. It was the beginning of a new era in graduate instruction in mathematics at Harvard, and mathematics has been taught here in that spirit ever since.

"These were the influences which moulded Bôcher's scientific life when he was yet an undergraduate; this was the atmosphere in which he lived. From here he went to Göttingen, where he studied for six semesters under Klein. An account of his work there will be found in the speaker's forthcoming article on "The Life and Services of Maxime Bôcher," in the *Bulletin of the American Mathematical Society*, May, 1919."

Mathematics Club of the University of Maine, Orono, Me.
[*1918*, 132, 453–454].

During the Fall Term, October to December, 1918, the club was inactive, since the University of Maine was given over to the work of the Student's Army Training Corps, and regular university work was suspended. Many of the members most active in the club last year did not return on account of graduation or war conditions.

In January, 1919, the club was reorganized and the following officers elected. President, Grace H. Hodgdon '19; vice-president, Flavia L. Richardson '20; secretary-treasurer, Edith I. Deering '21; faculty member of the program committee, Professor Myron O. Tripp.

Programs since January, 1919, have been as follows:

January: Social meeting at the home of Dean James N. Hart.

February: "The training of teachers for mathematics" by Professor Myron O. Tripp; "War Savings Stamps as an investment" by Flavia L. Richardson '20.

March: No meeting was held in March.

The Mathematical Club of the University of Nebraska, Lincoln, Nebr.
[*1918*, 313–315].

At the University of Nebraska, as at many other universities, the mathematics club was inactive during the first semester on account of war activities. Soon after the opening of the second semester, the club resumed its activities under the leadership of the officers given below.

Officers, 1919: President, Frances R. Botkin '19; secretary, Mervyn C. Kimberly '20; faculty member of executive committee, Professor Albert Babbitt.

February 13, 1919: "The game of Nim"[1] by Professor Meyer G. Gaba.

[1] A brief discussion of the game of Nim, with references to related literature, was included in Club Topic No. 7, published in the March, 1918, issue of this Monthly (Vol. XXV, pp. 139–142).

March 20: "Complex numbers" by Josiah Brooks '21; "Roots of unity" by Paul J. White '21.

FLATLANDERS—A MATHEMATICAL PLAY IN ONE ACT.[1]

Persons in the Play.

MR. CUBE.
MASTER RATIO—A Schoolmaster, inclined to be fanatical.
CYCLUS—A young nobleman just entering school.
BARON MULTILATUS—Father of Cyclus.

Prologue.

Spoken by Cube.

Dear People of the Third Dimension,
I have to bring to your attention
A place that's hard to understand—
A country that is called *Flatland.*
The people here, as you will see,
Are long or wide—as the case may be,
But one thing they are wont to slight—
They never heard of having *height!*
That woman is a fashion-plate
Whose form's a line that's thin and straight.
And, lest some man should fail to see
A line that's drawn so daintily
And e'er he'd time to step aside
With horrid bump they should collide.
She hums a note that's thin and clear
To let him know she's drawing near.
Since Flatlanders' nobility
Is *sides* instead of ancestry
The King's a circle, and his Prides
Are polygons of many sides.
The Triangle's prestige is small.
The Angle has no name at all.
And yet, unto a Cube like me
The case is grievous as can be.
For, though I'll fight 'gainst circumstance
To pull them *up* from ignorance,
I fear in flight they may not revel—
For Flatlanders are on the *level*!

[1] Performed before The Mathematics Club of Vassar College, February 20, 1919. The Prologue and Epilogue were written by Kathleen Millay '21, and the Scene was adapted by Lucile Free '21. This adaptation of E. A. Abbott's *Flatland* was suggested by a description of a dramatization, in seven scenes, performed at the Haberdashers' Aske's Girls' School, Acton, England, in June, 1913. The description in *School Science and Mathematics*, October, 1914, volume 14, pp. 583–587, was reprinted from *The Mathematical Gazette*, January, 1914, volume 7, pp. 228–231.

The Scene.

Setting: The schoolroom. Around the walls are hung various shaped cardboard figures—inhabitants of Flatland. There is the Teacher's desk, and the benches for the children. In the front row of these are—in line—a cardboard: Triangle; Pentagon; Straight Line; Octagon. Problems treating Areas are on the blackboard. Cyclus is just entering the school. Ratio enters on the other side.

The costumes may be all black and white with cardboard printed names hung around the necks of the characters. Cube may be inside a paper cube if desired.

Time: To-day, in Flatland.

Ratio. Good-Morning, Cyclus. So this is our new pupil.

Cyclus. Good-morning, Master Ratio. Father said you were to teach me my angles very well this year so that I will not make the mistake of associating with people beneath my rank.

Ratio. My students always know their angles very well. But first, let's see how much you know already. You look like a bright lad, and doubtless the son of the great Baron Multilatus has had practice in discovering the rank of his playfellows. Begin over there on the first row, young man, and tell me the class of each child.

Cyclus [*Goes to cardboard figures*]. This, this—let me see—why, this is a Triangle! How stupid of you to admit Triangles to such an exclusive school. I'm sure my father won't approve of it. And this next one—this is a Pentagon.

Ratio. Right, sir.

Cyclus. And this—oh, this is only a woman. I won't even have to bother with it. But this next person seems to trouble me quite a good deal. It couldn't be! Why, yes, you're a Decagon. I would like to walk home from school with you sometime, Mr. Decagon. I'm sure we'll have very much in common. Your father must be a Count.

Ratio. That will do, Cyclus. I see where your trouble lies. By the way, that last gentleman was not a Decagon, but an Octagon. I'm sure we can correct your faults very easily. Now, sir, have you any questions before we begin the lesson for the day?

Cyclus. Please, sir, what is a straight line?

Ratio. A Straight Line is formed by a moving Point.

Cyclus. And what is an Area?

Ratio. An Area is formed by moving a Straight Line.

Cyclus. [*Musing to himself.*] Then I wonder why they couldn't move to the right or left.

Ratio. I beg pardon, sir. You wonder why *who* couldn't move to the right or left?

Cyclus. I was just thinking about a very strange dream I had last night. I was in a strange land. The people all seemed to me to be very unhappy because they were so crowded together. They could not run about and play as we do, but had to move all the time in a Straight Line. They could never pass each other, but when they met all they could do was to go back again the way they had come. It was all so tiresome and stupid. I insulted their king by taking

him for a woman, although how they could tell him from anyone else is more than I can see. A funny Straight Line bumped in to me, and I told him to go to the right. He looked at me in a blank, helpless sort of way, and began to back up. Master Ratio, what do you suppose was the matter?

Ratio. That was a very peculiar dream, Cyclus, but it is all quite clear to me. You see those people only knew one dimension. How much happier they could have been if they had only known that they could move to the right or left in areas. Your dream proves to you, sir, what a lucky person you are. Your life is not hindered by ignorance of the possibilities you have.

Cyclus. [*Hesitating.*] But, Master Ratio, sometimes I think I *am* hindered. I wonder what would happen if you should move an Area.

Ratio. [*Calmly.*] Nothing at all would happen. See, I will illustrate for you. [*He pulls a piece of paper around on the top of his desk.*] There, I have moved it to the right and it is still in the same shape and size. And now to the left—backward,—forward. You cannot change an area. [*There is a knock at the door.*] Come in! [*Cube enters. Master Ratio goes up and feels of or inspects his angles. Of course, all he can see of the cube is a cross section.*] Ah, yes, Mister Square, How do you do? And what can I do for you today?

Cube. I am not a Square; I am a Cube.

Ratio. A what?

Cube. A Cube.

Ratio. And where do you come from?

Cube. I come from the Land-of-the-Third-Dimension.

Ratio. Third Dimension! What a peculiar language you have. According to the meaning of our word *Dimension* there is no third Dimension. And you call yourself a Cube. And yet, you are exactly what we call a Square.

Cube. My language is just the same as yours, Master Ratio, only the range of my vocabulary is much larger. And I tell you again that I am not a Square. If you take a Square and move it up, you will discover what I am.

Ratio. Up?—Up?—Now what can *up* mean? Not this way.—[*Illustrates again pulling his paper over the top of his desk, right, left,* etc.]—nor this, this or this. Cyclus, get me the dictionary, for I must find this word. And what do you mean, Mister Cube, when you say *Third* Dimension?

Cyclus. Maybe we're as ignorant as the people in my dream, after all.

Ratio. [*Irritated.*] No, no, Cyclus! What an absurd idea! What would your father, magistrate of our village, say if he knew you entertained such thoughts! No, no! [*Ratio and Cyclus search dictionaries. Cube inspects figures on the wall. Baron Multilatus enters without knocking.*] What a fruitless search. *Up* is not in any of our dictionaries.

Baron Multilatus. [*Sarcastically.*] Well, well, Master Ratio! I thought I should some day catch you unawares! Of course, *up* is not in the dictionaries. There is no such word. Do you think that you can put any two letters together and make a word? Why not look for *m-i*, or *k-t*, or *p-b*, *l-l*, anything instead of using *u-p*.

Ratio. But Mister Cube, this fellow here whom we commonly call Mister Square, wants me to move a Square *up* and find the Third Dimension. Baron! [*Dramatically.*] I feel as though I were on the point of making some wonderful new discovery! Something that will help our future generations to live freer, happier lives than ours have been. Something that will ——

Baron Multilatus. Enough of this, my man! I fear you are becoming mentally unbalanced. You'll only end by being burned as a heretic. And if you continue in this fanatical idea of yours I myself shall see that you get your just punishment. Come along, Cyclus—We'll seek a better master, and a saner one.

[*Exeunt Baron Multilatus with Cyclus.*]

Ratio. [*Musing.*] He thinks I'm crazy, does he? Well, well, perhaps I am,—but we'll see. We'll see. [*To Cube.*] And now, friend Cube, I'm off to my fellow-masters to see if I can get any help in this miraculous problem you have suggested. Doubtless they, too, will think me mad. Oh, when will the world learn to respect a man who is seeking after new truths? But—I'll meet you later, sir, and perhaps then we can bring happiness to my poor people. [*Exit Ratio.*]

Epilogue.

Spoken by Cube.

And thus, my friends, this demonstration
Portrays a grievous situation—
A people who would like to find
A way to educate the mind,
And yet, who miss inevitably
A truth we know quite naturally.
And, sometimes, when I contemplate
Their ignorance so desolate
I wonder if *we* fail to see
Some evident reality.
And so, I bring to your attention
The subject of the Fourth Dimension.

PROBLEMS AND SOLUTIONS.

EDITED BY B. F. FINKEL AND OTTO DUNKEL.

Send all communications about problems to B. F. FINKEL, Springfield, Mo.

PROBLEMS FOR SOLUTION.

2776. Proposed by C. P. SOUSLEY, State College, Penn.

Prove by elementary geometry that the Wallace lines of the extremities of any diameter of the circumscribed circle of a triangle intersect at right angles on the nine-point circle of the triangle.

2777. Proposed by W. D. CAIRNS, Oberlin College.

Prove that the two series

$$1 + \frac{\pi^4}{2^4 \cdot 4!} + \frac{\pi^8}{2^8 \cdot 8!} + \cdots,$$

and

$$\frac{\pi^2}{2^2 \cdot 2!} + \frac{\pi^6}{2^6 \cdot 6!} + \frac{\pi^{10}}{2^{10} \cdot 10!} + \cdots$$

are equal.

2778. Proposed by WARREN WEAVER, University of Wisconsin.

A partition of space is effected by means of five planes, none of which are parallel and no four of which pass through the same point, and six spheres. This divides all space into n regions, some of which are finite and some infinite. Considering it equally probable that a bird be in any one of the n regions show that the probability of its being caught (that is, of its being in one of the finite regions) is equal to or less than 78/99.

2779. Proposed by J. L. RILEY, Junior Agricultural and Mechanics College, Stephenville, Texas.

A parabola is placed with its axis horizontal; find the straight line of shortest descent from the curve to the focus.

406 (Algebra) [March, 1914]. Proposed by S. A. COREY, Albia, Iowa.

Solve the system of equations:

$$(1 - x)(a_1 + a_2y + a_3z) = d, \quad (1 - y)(b_1 + b_2x + b_3z) = g, \quad (1 - z)(c_1 + c_2x + c_3y) = h.$$

411 (Algebra) [April, 1914]. Proposed by V. M. SPUNAR, Chicago, Ill.

Determine $x_1, x_2, x_3, \cdots x_p$, from the equations:

$$\begin{aligned}
x_1 + x_2 + x_3 + \cdots + x_p &= a_0, \\
b_1x_1 + b_2x_2 + b_3x_3 + \cdots + b_px_p &= a_1, \\
b_1^2x_1 + b_2^2x_2 + b_3^2x_3 + \cdots + b_p^2x_p &= a_2, \\
\cdots\cdots\cdots

$$\frac{\int\cdots\int_T dx_1\cdots dx_n}{\int\cdots\int_S dx_1\cdots dx_n};$$

in other words, find the magnitude of a "solid angle" in n dimensions, with the "sphere" as unit solid angle.

Note. This problem was discussed and left unsolved by Schläfli in the *Quarterly Journal of Mathematics* for 1858, 1860, 1867.—Editor.

349 (Calculus) [December, 1913]. Proposed by C. N. SCHMALL, New York City.

If $y = a\cos(\log x) + b\sin(\log x)$, eliminate the constants a and b and obtain the equation

$$x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0.$$

198 (Number Theory) [November, 1913]. Proposed by the late ARTEMAS MARTIN.

Prove that every even number is the sum of two prime numbers.

Note. This problem has long been known and no proof has ever been given.—Editor.

201 (Number Theory) [December, 1913]. Proposed by E. T. BELL, University of Washington.

Eisenstein proposed (*Crelle*, t. 27, p. 282), as the simplest of several problems: "In the expansion of

$$\frac{1 + z + z^2 + \cdots + z^{p-1}}{(1-z)^{p-1}} - 1,$$

where p is prime, to show that the coefficients of the various powers of z are all divisible by p."

SOLUTIONS OF PROBLEMS.

231 (Number Theory) [April, 1915]. Proposed by A. J. KEMPNER, University of Illinois.

Is the series whose terms are the reciprocals of all positive integers not containing a given combination of figures, for example not containing the combination 37, convergent or divergent? Numbers such as $\frac{1}{37}$, $\frac{1}{370}$, $\frac{1}{5371}$ shall be omitted, numbers such as $\frac{1}{73}$, $\frac{1}{307}$, $\frac{1}{5317}$ shall be admitted as terms of the series. (Compare AMERICAN MATHEMATICAL MONTHLY, Volume 21, page 123.)

SOLUTION BY FRANK IRWIN, University of California.

Let $f(n)$ be the number of numbers in the class, g_n, of numbers of n digits that do not contain the given combination, for example 37. We can get these numbers by taking any number of g_{n-1} and adding a digit at the end; except that this digit must not be 7 if the number chosen from g_{n-1} ends with a 3. Since the number of these rejected numbers of g_{n-1} is evidently $f(n-2)$, we have the formula: $f(n) = 10f(n-1) - f(n-2)$. In the general case, where the given combination consists of k digits, we should get similarly, for $n > k$,

$$f(n) = 10f(n-1) - f(n-k). \tag{1}$$

(An exception would arise for such a combination as 37537; here, for instance, the formula $f(8) = 10f(7) - f(3)$ would not hold, for the number 375 belonging to g_3 would have been subtracted, as it should not have been, since the number obtained by writing after it 3753, viz., 3753753, does not belong to g_7, and so has not been counted in $f(7)$. It will be seen that such cases arise when the first l digits of the given combination are the same as the last l, $l < k$. We shall call such combinations "improper" and deal with them separately.)

From (1) we have $f(n-1) > f(n)/10$. Also since $f(n-1) = f(n)/10$, $n < k$, and $f(n-1) \geqq f(n)/10$, $n = k$ (= if the given combination begins with a zero), we have, for $n > k$, the string of relations $f(n-2) \geqq f(n-1)/10$, $f(n-3) \geqq f(n-2)/10$, $\cdots$. From these last inequalities we derive $f(n-k) \geqq f(n-1)/10^{k-1}$. Comparing this with (1), we see that $f(n) \leqq 10f(n-1) - f(n-1)/10^{k-1}$, or $f(n) \leqq 10rf(n-1)$, if we put $(10^k - 1)/10^k = r$. Since the reciprocal of any number in g_n is $\leqq 1/10^{n-1}$, it follows that the sum of the reciprocals of the numbers in $g_k, g_{k+1}, \cdots$ are less than the corresponding terms of the series

$$\frac{f(k)}{10^{k-1}}(1 + r + r^2 + \cdots).$$

This series converges, for r is < 1; and hence our given series converges.

The case of "improper" combinations may be rapidly dealt with by the following device. Write the digit a, k times at the end of the given combination, where a is any digit other than the first digit of the combination. This certainly gives us a "proper" combination (of $2k$ digits), so that, as already proved, the series formed from the harmonic series by omitting terms whose denominators contain this combination converges. But this series evidently contains all terms of the given series; this latter, then, converges too.

In like manner we may prove that *the series obtained by omitting terms containing all of a given set of combinations (some of which may be repeated) converges.* Here we may compare the series with that obtained by omitting terms containing the single combination formed from the given combinations by juxtaposition.

This supplies another proof of the proposition in my paper on "A Curious Convergent Series" in this MONTHLY for May, 1916, that proposition being the special case of the above in which each of the combinations consists of a single digit.

The proposition stated by Professor Moulton in his solution of Algebra 453 in this MONTHLY for Oct., 1916, refers, I suppose—the proof is not given in full—not to combinations of the kind here considered, but to selections, and without any reference to the juxtaposition of the digits of the selection in the omitted terms.

239 (Number Theory) [March, 1916; March, 1919]. Proposed by HAROLD T. DAVIS, Colorado Springs, Colorado.

Give a general method for determining the solution in integers of the equation

$$x^r - 10xy - (n + 1) + y = 0,$$

where r and n are positive integers.

II. SOLUTION BY FRANK IRWIN, University of California.

If $x = 0$, $y = n + 1$; noting this solution, we shall suppose in what follows $x \neq 0$.

The degree in x of the given equation may be reduced by the following device. $y - (n + 1)$ must be divisible by x, say $y - (n + 1) = y_1x$. Then our equation reduces to

$$x^{r-1} - 10xy_1 - 10(n + 1) + y_1 = 0.$$

Putting $y_1 - 10(n + 1) = y_2x$, $y_2 - 10^2(n + 1) = y_3x$, etc., we finally reach the equation

$$1 - 10xy_r - 10^r(n + 1) + y_r = 0, \tag{1}$$

where y and y_r are connected, it will be found, by the relation

$$y = (n + 1)[1 + 10x + 10^2x^2 + \cdots + 10^{r-1}x^{r-1}] + y_rx^r,$$

or

$$y = (n + 1)\frac{10^rx^r - 1}{10x - 1} + y_rx^r. \tag{2}$$

Rewriting (1) as

$$(10x - 1)y_r = 1 - 10^r(n + 1), \tag{3}$$

we see that x and y_r must have opposite signs.

(*i*) Let y_r be negative, $y_r = -y_r'$. Then (3) becomes $(10x - 1)y_r' = 10^r(n + 1) - 1$. Here the number on the right ends with the digit 9, as does also the first factor on the left. Hence we see that y_r' must end with 1. We get, then, a solution by taking for y_r' any factor, ending with 1, of the right side, and determining x and y from (3) and (2).

(*ii*) Let x be negative, $x = -x'$. Then (3) becomes

$$(10x' + 1)y_r = 10^r(n + 1) - 1.$$

We take y_r equal to any factor of the right side that ends with 9.

This gives us all solutions.

Suppose that $10^r(n + 1) - 1$ can be factored as $a \cdot b$, where a ends with 1, b with 9. Then

we may take either $y_r = -a$, whence $x = (b+1)/10$; or $y_r = b$, $x = (-a+1)/10$. Thus solutions go in pairs, the x of either solution being obtained by adding 1 to the y_r of the other and dividing by 10. There are, then, always an even number of solutions, and always at least two, viz., those where $y_r = -1$, $x = 10^{r-1}(n+1)$; $y_r = 10^r(n+1) - 1$, $x = 0$.

Finally it may be noted that the more general equation

$$ax^r - bxy + y - c = 0$$

may be solved by similar methods. Here we should have to pick out such divisors of $b^r c - a$ as are $\equiv \pm 1 \pmod{b}$. It may be interesting to apply the method to an example. Let us solve

$$x^4 - 10xy - 22 + y = 0.$$

We have to solve

$$(10x-1)y_4 = -219999 = -3 \times 13 \times 5641$$

(5641 prime). The values of y_4 with the corresponding solutions of our equation are:

$y_4 = -5641$, $x = 4$, $y = 6$; $y_4 = -1$, $x = 22000$ (y a very large number);

$y_4 = 39$, $x = -564$, $y = -17937434$; $y_4 = 219999$, $x = 0$, $y = 22$.

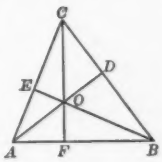
247 (Number Theory) [June, 1916]. Proposed by NORMAN ANNING, Chilliwack, B. C.

To dissect the triangle whose sides are 52, 56, 60 into three Heronian triangles by lines drawn from the vertices to a point within.

The word Heronian is used in the sense of the German Heronische (Wertheim, *Anfangsgründe d. Zahlenlehre*, p. 140) to describe a triangle whose sides and area are integral.

SOLUTION BY FRANK IRWIN, University of California.

The orthocenter, O, may be taken as the required point. Let ABC be the triangle with $a = 60$, $b = 52$, $c = 56$; and let the feet of the perpendiculars from A, B, C on the opposite sides be D, E, F, respectively. Then the various lines in the figure, calculated as indicated below, are: $BD = 168/5$, $DC = 132/5$, $CE = 396/13$, $EA = 280/13$, $AF = 20$, $FB = 36$; $AO = 25$, $BO = 39$, $CO = 33$. Finally, area $BOC = 594$, area $COA = 330$, and area $AOB = 420$; so that the sides and areas of these three triangles are integral, as asserted.



The explanation of these facts depends on the following proposition: If the sides and area of the triangle ABC are rational, the same is true of the triangles BOC, COA, AOB. (Then by multiplying the dimensions of the figure by a suitable integer everything can be made integral.) For the three altitudes are rational, as also the radius r of the inscribed circle (since rs = area). Thus $\tan A/2$ is rational, and so, then, are $\cos^2 A/2$ and $\cos A$. Therefore, $AF = b \cos A$ is rational, and similarly, FB, BD, etc. Then the triangle AOF is rational (that is, has rational sides), since one of its sides, AF, is rational, and it is similar to the rational triangle ABD.

2678 [February, 1918]. Problem proposed by C. F. GUMMER, Queen's University, Canada.

Find necessary and sufficient conditions that the roots of the equation $x^{n+1} + a_1 x^n + a_2 x^{n-1} + \cdots + a_{n+1} = 0$ may be all real and separated by the roots of $x^n + b_1 x^{n-1} + b_2 x^{n-2} + \cdots + b_n = 0$.

SOLUTION BY THE PROPOSER.

Consider the equations

$$(1) \qquad f(x) \equiv x^{n+1} + a_1 x^n + \cdots = 0,$$

$$(2) \qquad g(x) \equiv x^n + b_1 x^{n-1} + \cdots = 0,$$

$$(3) \qquad R_1(x) \equiv c_0 x^{n-p} + c_1 x^{n-p-1} + \cdots = 0,$$

$$(4) \qquad R_2(x) \equiv d_0 x^{n-p-q} + d_1 x^{n-p-q-1} + \cdots = 0,$$

. . . . . .

where $R_1(x)$ is the remainder with sign changed on dividing $f(x)$ by $g(x)$, $R_2(x)$ has the same relation to $g(x)$ and $R_1(x)$, etc.

That the roots of (1) may be real and separated by those of (2), it is necessary and sufficient that

(*a*) the roots of $g(x)$ shall be real and distinct (such as $\beta_1 < \beta_2 < \cdots < \beta_n$) and

(*b*) $f(-\infty), f(\beta_1), \cdots, f(\beta_n), f(\infty)$ shall have alternate signs.

Now $f(\beta_i) = -R_1(\beta_i)$, and $R_1(x)$ cannot change sign more than $n-1$ times. Hence, (*a*) and (*b*) are equivalent to the conditions that c_0 shall be positive, and that the roots of (2) are all real and separated by those of (3) (implying that $p = 1$). These are in turn equivalent to the conditions that c_0, d_0 shall be positive, $p = q = 1$, and the roots of (3) real and separated by those of (4), and so on.

Hence, a set of necessary and sufficient conditions is

$$p = q = \cdots = 1;\ c_0, d_0, \cdots, \text{all positive.}$$

It may be shown that these are equivalent to the conditions

$$\begin{vmatrix} 1 & b_1 & b_2 \\ 1 & a_1 & a_2 \\ 0 & 1 & b_1 \end{vmatrix}, \quad \begin{vmatrix} 1 & b_1 & b_2 & b_3 & b_4 \\ 1 & a_1 & a_2 & a_3 & a_4 \\ 0 & 1 & b_1 & b_2 & b_3 \\ 0 & 1 & a_1 & a_2 & a_3 \\ 0 & 0 & 1 & b_1 & b_2 \end{vmatrix}, \quad \cdots$$

all positive. For the calculation of the remainders see papers by E. B. Van Vleck (*Annals of Mathematics*, 1899, pp. 1–13) and A. J. Pell and R. L. Gordon (*Annals of Mathematics*, June, 1917, pp. 188–193).

2701 [May, 1918]. Proposed by E. H. WORTHINGTON, University of Pennsylvania.

Find the sum of the infinite series

$$\frac{1}{5} + \frac{1\cdot 2}{5\cdot 7} r + \frac{1\cdot 2\cdot 3}{5\cdot 7\cdot 9} r^2 + \cdots + \frac{n!}{5\cdot 7\cdot 9 \cdots (2n+3)} r^{n-1} + \cdots$$

Verify your result for $r = 0$ and $r = 1$.

SOLUTION BY A. M. HARDING, University of Arkansas.

It can be easily shown that the following equation holds for all integral values of $n \geqq 0$:

$$\int_0^1 t^{1/2}(1-t)^{n+1}dt = \frac{2(n+1)}{2n+5} \int_0^1 t^{1/2}(1-t)^n dt.$$

Multiplying both sides by x^n and substituting $n = 0, 1, 2, \cdots (n-1)$, gives

$$\frac{3}{4}\int_0^1 t^{1/2}(1-t)dt = \tfrac{1}{5},$$

$$\frac{3}{4}\int_0^1 t^{1/2}(1-t)^2 x dt = \frac{1\cdot 2}{5\cdot 7} 2x,$$

$$\frac{3}{4}\int_0^1 t^{1/2}(1-t)^3 x^2 dt = \frac{1\cdot 2\cdot 3}{5\cdot 7\cdot 9} (2x)^2,$$

. . . . . .

$$\frac{3}{4}\int_0^1 t^{1/2}(1-t)^n x^{n-1} dt = \frac{n!}{5\cdot 7\cdot 9 \cdots (2n+3)} (2x)^{n-1}.$$

. . . . . .

Letting $x = r/2$ and adding, we have

$$\frac{3}{4}\int_0^1 t^{1/2}(1-t)[1 + (1-t)x + \cdots + (1-t)^{n-1}x^{n-1} + \cdots]dt$$
$$= \frac{1}{5} + \frac{1\cdot 2}{5\cdot 7} r + \cdots + \frac{n!}{5\cdot 7\cdot 9 \cdots (2n+3)} r^{n-1} + \cdots.$$

The series in the right member of this equation converges if $r < 2$. In this case, the series under the integral converges uniformly to the sum $\dfrac{1}{1-(1-t)r/2}$. Hence, the sum of the given series is given by

$$S = \frac{3}{2}\int_0^1 \frac{t^{1/2}(1-t)}{2-(1-t)r}\,dt.$$

When $r = 0$, we find $S = 1/5$, and when $r = 1$, the integral reduces to $5 - (3\pi/2)$; we do not know of any evaluation of the series independent of the integral calculus.

Also solved by H. F. GUMMER and E. H. CLARKE.

2709 [June, 1918]. Proposed by E. V. HUNTINGTON, Harvard University.

The following problem was suggested to the proposer by a professor of biology, who has found the result useful in certain problems concerning the equilibrium of chemical reactions. Starting with

$$\mu(c_1 + y - x)(c_2 - x)(c_3 - x)\cdots(c_n - x) = \lambda(b_1 + x)(b_2 + x)(b_3 + x)\cdots(b_m + x),$$

where $\mu c_1c_2c_3\cdots c_n = \lambda b_1b_2b_3\cdots b_m$ (all the letters being positive), find the limit of x/y as y approaches zero; and show that for small values of y, the value of x/y is always *less* than this limit.

I. SOLUTION BY THE PROPOSER.

Let $y = f(x) = f(0) + f'(0)\cdot x + \frac{1}{2}f''(0)\cdot x^2 + \cdots$.

First. From the given equations, when $x = 0$, $y = 0$; hence $f(0) = 0$.

Second. Taking logarithms and differentiating, we have

$$\frac{f'(x)-1}{c_1+y-x} = \left(\frac{1}{c_2-x}+\frac{1}{c_3-x}+\cdots+\frac{1}{c_n-x}\right)+\left(\frac{1}{b_1+x}+\frac{1}{b_2+x}+\frac{1}{b_3+x}+\cdots+\frac{1}{b_m+x}\right),$$

whence

$$\frac{f'(0)}{c_1} = \left(\frac{1}{c_1}+\frac{1}{c_2}+\frac{1}{c_3}+\cdots+\frac{1}{c_n}\right)+\left(\frac{1}{b_1}+\frac{1}{b_2}+\frac{1}{b_3}+\cdots+\frac{1}{b_m}\right).$$

Third. Differentiating again, we have

$$\frac{(c_1+y-x)f''(x)-[f'(x)-1]^2}{(c_1+y-x)^2} = \left[\frac{1}{(c_2-x)^2}+\frac{1}{(c_3-x)^2}+\cdots+\frac{1}{(c_n-x)^2}\right] - \left[\frac{1}{(b_1+x)^2}+\frac{1}{(b_2+x)^2}+\frac{1}{(b_3+x)^2}+\cdots+\frac{1}{(b_m+x)^2}\right],$$

whence

$$\frac{f''(0)}{c_1} = \left[\left(\frac{1}{c_2}+\frac{1}{c_3}+\cdots+\frac{1}{c_n}\right)+\left(\frac{1}{b_1}+\frac{1}{b_2}+\frac{1}{b_3}+\cdots+\frac{1}{b_m}\right)\right]^2 + \left[\frac{1}{c_2^2}+\frac{1}{c_3^2}+\cdots+\frac{1}{c_n^2}\right]-\left[\frac{1}{b_1^2}+\frac{1}{b_2^2}+\frac{1}{b_3^2}+\cdots+\frac{1}{b_m^2}\right],$$

which is clearly *positive*.

Fourth. With these values of $f(0)$, $f'(0)$, and $f''(0)$, we have

$$\frac{y}{x} = f'(0) + \frac{f''(0)}{2}\cdot x + \cdots, \qquad \text{or} \qquad \frac{x}{y} = \frac{1}{f'(0)} - \frac{f''(0)}{2[f'(0)]^2}\cdot x + \cdots,$$

from which the required solution is obvious.

Note: It is clear that x and y have like signs for small values of x since $f'(0) > 0$.

II. SOLUTION BY A. M. HARDING, University of Arkansas.

Solving the given equation for y gives

$$y = x - c_1 + \frac{\lambda(b_1+x)(b_2+x)\cdots(b_m+x)}{\mu(c_2-x)(c_3-x)\cdots(c_n-x)} = x - c_1 + \frac{c_1(1+x/b_1)(1+x/b_2)\cdots(1+x/b_m)}{(1-x/c_2)(1-x/c_3)\cdots(1-x/c_n)}.$$

Let c_k denote the smallest of $c_2, c_3, \cdots c_n$. Then, if $x < c_k$, each of the expressions $\dfrac{1}{1-x/c_i}$, $(i = 2, 3, \cdots, n)$, may be expanded into a convergent series and we obtain, after reduction,

$$y = Ax + Bx^2 + Cx^3 + Dx^4 + \cdots, \tag{1}$$

where

$$A = 1 + c_1 \sum_{i=1}^{m} \frac{1}{b_i} + c_1 \sum_{i=2}^{n} \frac{1}{c_i} = c_1 \left[\sum_{i=1}^{m} \frac{1}{b_i} + \sum_{i=1}^{n} \frac{1}{c_i} \right]$$

and $B, C, D, \cdots$ are all positive.

Since each of the series formed from $\frac{1}{1 - x/c_i}$ converges in the interval $0 \leqq x < c_k$, the power series (1) will converge in this interval. And, since in (1) A is not zero, there is one and only one solution for x in terms of y, and this solution may be obtained as a power series in y convergent for sufficiently small values of y.

Let

$$x = \alpha y + \beta y^2 + \gamma y^3 + \delta y^4 + \cdots. \tag{2}$$

Substitute (2) in (1), equate coefficients of like powers of y, and obtain

$$\alpha = \frac{1}{A}, \qquad \beta = -\frac{B}{A^3}, \qquad \gamma = \frac{2B^2}{A^5} - \frac{C}{A^4}, \cdots.$$

That is

$$x = \frac{1}{A} y - \frac{B}{A^3} y^2 + \left(\frac{2B^2}{A^5} - \frac{C}{A^4} \right) y^3 + \cdots$$

or

$$\frac{1}{A} - \frac{x}{y} = \frac{B}{A^3} y - \left(\frac{2B^2}{A^5} - \frac{C}{A^4} \right) y^2 + \cdots. \tag{3}$$

Thus

$$\lim_{y \to 0} \frac{x}{y} = \frac{1}{A} = \frac{1}{c_1 \left(\sum_{i=1}^{m} \frac{1}{b_i} + \sum_{i=1}^{n} \frac{1}{c_i} \right)}.$$

For sufficiently small values of y the right member of (3) is positive. Hence, for small values of y, the value of x/y is always less than $1/A$.

NOTES AND NEWS.

EDITED BY E. J. MOULTON, Northwestern University, Evanston, Ill.

Dr. G. H. LIGHT has been promoted from assistant professor to associate professor of mathematics at the University of Colorado.

The fellows in mathematics at the University of Chicago for next year are Miss GLADYS GIBBENS, Mrs. MAYME I. LOGSDON, and Mr. FRANK E. WOOD.

Dr. C. C. CAMP, who recently returned from war service in France, has been appointed instructor in mathematics at the University of Illinois.

At Brown University Captain R. W. BURGESS, now of the Bureau of Statistics, Washington, and Dr. R. E. GILMAN have been appointed assistant professors of mathematics, and Mr. C. R. ADAMS instructor.

At Cornell University Assistant Professor F. R. SHARPE has been promoted to a full professorship and Mr. V. G. GROVE, of the University of Kentucky, has been appointed instructor.

Dr. E. A. KIRCHER, now in the employ of the National City Bank, New York City, has been appointed instructor of mathematics at Yale University.

Mr. C. A. NELSON has been appointed instructor of mathematics at the University of Kansas.

Mr. A. D. CAMPBELL, now instructor in mathematics at Cornell University, has been appointed instructor at Yale.

Dr. G. W. MULLINS of Barnard College, Columbia University, has been promoted to an assistant professorship of mathematics.

At Harvard University, Assistant Professor G. D. BIRKHOFF has been promoted to a full professorship, Messrs. B. H. BROWN, C. A. RUPP, and R. S. TUCKER have been appointed instructors in mathematics, and Dr. I. A. BARNETT and Dr. H. C. M. MORSE have been appointed Benjamin Peirce instructors in mathematics. Professor H. N. DAVIS, assistant professor of physics since 1910, has been appointed professor of mechanical engineering.

At the Massachusetts Institute of Technology the following instructors in mathematics have been appointed for next year: Mr. RAYMOND DOUGLASS, of the University of Maine; Dr. J. S. TAYLOR, of the University of California, and Dr. NORBERT WIENER, of Harvard University. Assistant Professor H. C. BRADLEY of the department of drawing and descriptive geometry has been promoted to an associate professorship; in the same department Mr. S. A. BREED has been appointed instructor.

The following announcements come from the mathematics department of the University of Minnesota. Assistant Professor DUNAHM JACKSON, of Harvard, has been appointed to a professorship. Major W. L. HART has been appointed to an assistant professorship. Mr. R. W. BRINK has been promoted to an assistant professorship, and given leave of absence for a year to enable him to accept a lectureship at the University of Edinburgh. Dr. C. H. YEATON has been appointed as an instructor to take Mr. Brink's place for the year 1919–1920. Miss MINNA J. SCHICK has been appointed instructor for the first Quarter of 1919–1920, as a substitute for Professor G. N. BAUER who is absent on leave until January 1, 1920.

Professor F. E. MILLER, head of the department of mathematics in Otterbein College and a charter member of the Association, died March 26, 1919.

On March 28, Major J. L. COOLIDGE, of Harvard University, commenced a course of lectures (twice a week) at the University of Paris on "Geometry in the complex domain."

Of the six three-hour examinations held at Annapolis this month for the

purpose of selecting instructors of mathematics at the Academy, one was on theoretical mechanics as in Miller and Lilly's *Analytic Mechanics*, and another was on strength of materials and hydromechanics as in Smith's *Strength of Material* and Alger's *Hydromechanics*. The other examinations were on differential and integral calculus as in Granville's work, analytic geometry as in Smith and Gale's *New Analytic Geometry*, trigonometry as in Brown's *Trigonometry and Stereographic Projections*, and Wentworth, *College Algebra* (revised), 1902, and Wentworth's *Solid Geometry* (revised), 1909.

Higher Educational Circular No. 14 issued in February, 1919, by the Bureau of Education, Washington, gives an account of advanced educational work at the Bureau of Standards where, during the past ten years, graduate courses in physics, mathematics, and chemistry have been maintained. Seventeen men have used investigations made at the Bureau for their doctoral theses. Eleven of these have received credit for attendance upon the lecture courses at the Bureau. Courses requiring considerable mathematical development, such as advanced optics, and thermodynamics have been given; but the emphasis in these courses was primarily upon the physics. The following courses in pure and applied mathematics have been offered recently: 1917–18—Least squares by Dr. P. W. MERRILL, Bureau of Standards; Electrical oscillations by Dr. LOUIS COHEN, Signal Corps; Introduction to mathematical physics by P. G. AGNEW, Bureau of Standards; Electric waves by Professor W. S. FRANKLIN, Massachusetts Institute of Technology. 1918–19—Advanced differential equations by Major F. R. MOULTON, University of Chicago; Introduction to mathematical physics by Professor J. S. AMES, Johns Hopkins University.

In addition to the reports of summer courses in mathematics given in the March, April and May numbers of the MONTHLY, we have the following:

Leland Stanford University: June 17–August 30. Professor H. F. BLICHFELDT, Algebra (4 units); coördinate geometry (4 units); Reading course.

University of Pennsylvania: July 8–August 16. Professor G. H. HALLETT: Elementary algebra (to quadratics); plane trigonometry; analytic geometry; higher calculus. Professor H. H. MITCHELL: College algebra; differential calculus; integral calculus; mathematical theory of probability. Professor R. L. MOORE: Elementary algebra (quadratics and beyond); plane geometry; solid geometry; introduction to the theory of functions of a complex variable.

University of Michigan, June 29–August 2. Professor W. W. BEMAN: Geometry and Algebra for teachers (2 hours credit); differential equations (2 hours). Professor T. L. MARKLEY: Theory of functions of a complex variable (2 hours credit); algebra (2 hours); modern geometry and analytic geometry of three dimensions (2 hours). Professor L. C. KARPINSKI: Elementary algebra (for entrance); history of mathematics (2 hours credit). Professor C. J. COE: Algebra (2 hours credit); Analytic geometry (4 hours); Professor V. C. POOR: Integral calculus and differential equations (four or five hours credit). Dr. G. H. CRESSE: Plane Geometry (for entrance); plane trigonometry (two hours

credit). Dr. E. S. ALLEN: Solid geometry (for entrance). Dr. R. B. ROBBINS: Mathematical theory of statistics (two hours credit); introduction to the mathematical theory of interest (two hours); introduction to the mathematical theory of life insurance (two hours). Dr. A. L. NELSON: Elementary course in differential and integral calculus (four or five hours credit). Dr. L. J. ROUSE: Analytic geometry (continuation of Professor Coe's course—four hours credit).

In addition to those recorded in our last issue, as having been elected fellows of the National Academy of Sciences at the April meeting, mention should be made of Professor E. J. WILCZYNSKI, University of Chicago.

At the May meeting of the American Academy of Arts and Sciences, Professors JOSEPH LIPKA, G. A. MILLER, F. R. MOULTON, and VIRGIL SNYDER were elected Fellows in Class I, section 1—Mathematics and Astronomy.

To the Division of Physical Sciences of the National Research Council the American Mathematical Society elected Professors E. W. BROWN, L. E. DICKSON, and H. S. WHITE, as representatives; the American Physical Society elected Professor E. B. WILSON. Professor Wilson is a member of the Executive Committee of the Division.

At the seventeenth annual meeting of the Association of Teachers of Mathematics in New England held at Boston University, May 3, 1919, Miss FLORENCE P. LEWIS, exchange professor from Goucher College at Wellesley College, presented a paper entitled "History of the parallel postulate," and Professor C. L. BOUTON, of Harvard University, discussed "Photogrammetry." Professor W. R. RANSOM, of Tufts College, is president of the Association.

At the Educational Congress held under the direction of The University of the State of New York in Albany, May 19th to 28th, the papers read included the following by members of the Association: "Mathematical Requirements" by Professor J. W. YOUNG and President F. C. FERRY; "Mathematics in the Junior High School" by Mr. WILLIAM BETZ; "Mathematics of the Senior High School of the Future" by Professor H. E. HAWKES; "Experiments in Teaching Secondary Mathematics" by Miss VEVIA BLAIR; "Applied Mathematics in High School Courses" by Professor W. E. BRECKENRIDGE; "Projects for Mathematical Research" by Professor D. E. SMITH; "Training of Mathematics Teachers" by Professors R. C. ARCHIBALD and HARRY BIRCHENOUGH.

The Association of Teachers of Mathematics in the Middle States and Maryland held a joint meeting with the Association of Mathematics Teachers of New Jersey at Newark, N. J., on May 3. The program included: "Test of intelligence for admission to college," by Professor A. L. JONES, Columbia University; "Certain undefined elements and tacit assumptions in the first book of Euclid's Elements,"—the presidential address of the New Jersey Association—by Mr.

H. E. WEBB, Central High School, Newark, N. J.; "On Newton's method of approximation," by Dean H. B. FINE, Princeton University; "Some illustrations of statistical methods," by Mr. P. C. H. PAPPS, Newark, N. J.

The two hundred and third regular meeting of the American Mathematical Society at New York City was held on April 26, 1919, at Columbia University. Twenty-three papers of the usual research nature were presented. In addition to these papers, Professor DUNHAM JACKSON and Dr. T. H. GRONWALL gave reports on work in ballistics at Aberdeen and Washington. Attending members of the Society took luncheon and dinner together at the Faculty Club. A full report of the meeting will be found in the *Bulletin of the American Mathematical Society*.

SUMMER MEETING OF THE ASSOCIATION.

The fourth summer meeting of the Association will be held at the University of Michigan, Ann Arbor, Michigan, on Thursday and Friday, September 4–5, 1919. It will immediately follow the meeting of the American Mathematical Society to be held at the same place September 2–4. A joint meeting of the two organizations will be arranged for Thursday afternoon, and a joint dinner for Thursday evening. Detailed programs of the meeting will be mailed to all members of the Association at a later date, but it may be announced at this time that general arrangements have already been made for the accommodation of attending members. For this purpose, both the Helen Newberry Residence and the recently built Michigan Union Building will be available. The rates for rooms in Newberry Residence will be one dollar per day; in the Michigan Union Building, one dollar and fifty cents per day. Meals will be furnished under the auspices of the Michigan Union at reasonable rates. Special provisions will be made for ladies and married couples.

Ann Arbor is on the main line of the Michigan Central Railroad and of the Ann Arbor railroad.

The American Astronomical Association will meet in Ann Arbor probably from Monday to Wednesday of the same week.

University of Wisconsin

Summer Session, 1919

June 30 to August 8 (Law School, June 23 to August 29)

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THE MATHEMATICAL ASSOCIATION OF AMERICA now has over eleven hundred individual and institutional members. There are already nine sections formed, representing twelve different states. The Association has held so far two national meetings per year, one in September and one in December. The sections, for the most part, hold two meetings each year. All meetings, both national and sectional, are reported in the Official Journal, and many of the papers presented at these meetings are published in full.

The slogan of the Association is to include in its membership every teacher of collegiate mathematics in America and to make such membership worth while. Application blanks for membership may be obtained from the Secretary, W. D. Cairns, 27 King Street, Oberlin, Ohio.

THIRD ANNUAL MEETING OF THE KENTUCKY SECTION.[1]

The Third Annual Meeting was held at the University of Kentucky, Saturday, May 17, 1919, Professor E. L. REES presiding. The following papers were read: "A chart of mathematical history" by E. L. Rees, Univ. of Kentucky; "A problem in linkages" by H. P. Pettitt, Univ. of Kentucky (by invitation); "What is a curve?" by C. H. Richardson, Georgetown College (by invitation); "A problem in maxima and minima" by H. H. Downing, Univ. of Kentucky; "Syzygies obtained from the associative law for $n = 2$" by G. W. Smith, Univ. of Kentucky; "The catenoid the only minimal surface of revolution" by W. W. Elliott, Univ. of Kentucky; "Present day tendencies in the analytics course" by P. P. Boyd, Univ. of Kentucky; "Review of Shaw's *Philosophy of Mathematics*" by J. M. Davis, Univ. of Kentucky.

Professor C. G. CROOKS, Center College, Danville, Ky., was elected chairman and Dr. G. W. SMITH, University of Kentucky, was elected secretary-treasurer. After the meeting the members were the guests of the retiring chairman, Professor REES, at a very delightful luncheon served in the rooms of the University Cafeteria.

H. H. DOWNING, *Secretary-Treasurer.*

THE NATIONAL COMMITTEE ON MATHEMATICAL REQUIREMENTS.

The General Education Board has appropriated the sum of sixteen thousand dollars for the use of the National Committee on Mathematical Requirements. This Committee was appointed some three years ago by the Mathematical Association of America for the purpose of giving national expression to the movement for reform in the teaching of mathematics in secondary schools and colleges, which movement had for many years been actively furthered by various sectional organizations of teachers throughout the country.

According to plans adopted, one college man and one secondary school man are to devote their whole time to the work of the Committee for one year beginning July 1, 1919. Professor J. W. YOUNG, of Dartmouth College, has been selected as the college man in question. He will act as chairman and treasurer of the Committee and will have general charge of the work with headquarters in Hanover, N. H. Mr. J. A. FOBERG, of the Crane Technical High School of Chicago, is the representative of the secondary schools who will devote his whole time to work of the Committee. He will act as vice-chairman and secretary of the Committee with headquarters in Chicago. The other members of the Committee are Professors A. R. CRATHORNE, of the University of Illinois,

[1] This section was formerly the Mathematics Section of the Association of Kentucky Colleges which held eight annual meetings.